INDIAN STATISTICAL INSTITUTE

Students' Brochure

PART II

Master of Statistics

(Effective from 2015-16 Academic Year)

(See PART I for general information, rules and regulations)

The Headquarters is at

203 BARRACKPORE TRUNK ROAD

KOLKATA 700108

INDIAN STATISTICAL INSTITUTE

Master of Statistics

Contents

1 Curriculum

The two-year programme consists of a total of twenty credit-courses distributed over four semesters. The curriculum of the first year is divided into the two following streams.

- B-stream (for B.Stat. graduates),
- NB-stream (for other students).

These two streams merge in the second year, where the curriculum is divided into the six following specializations/tracks, one of which has to be chosen by the student before the beginning of the third semester.

- Probability specialization
- Theoretical Statistics specialization
- Applied Statistics specialization
  - Actuarial Statistics track
  - Biostatistics track
  - Computational Statistics track
  - Finance track

Apart from the courses described in the next section and Introductory Computer Programming for NB-stream, all courses are allocated four hours of classes per week.

First Year: B-Stream

Semester I	Semester break	Semester II
Statistical Inference I	Training	Large Sample Statistical Methods
Regression Techniques		Measure Theoretic Probability
Multivariate Analysis		Resampling Techniques
Stochastic Processes		Elective course
Categorical Data Analysis		Elective course

Elective courses for B-stream in First Year, Second Semester

Metric Topology and Complex Analysis (prerequisite for Probability Specialization)

Abstract Algebra

Optimization Techniques

Sample Surveys and Design of Experiments (prerequisite for Applied Statistics Specialization)

First Year: NB-Stream

Semester I	Semester break	Semester II
Statistical Inference I	Training	Large Sample Statistical Methods
Linear Algebra and Linear Models		Multivariate Analysis
Probability Theory		Elective course
Analysis I		Elective course
Regression Techniques		Elective course
Introductory Computer Programming (non-credit)*		

*This course may be spread over two semesters also. A final examination is not mandatory for this course.

Elective courses for NB-stream in First Year, Second Semester

Nonparametric and Sequential Methods (prerequisite for Theoretical Statistics Specialization and Applied Statistics Specialization)

Measure Theoretic Probability (prerequisite for Theoretical Statistics Specialization and Probability Specialization)

Analysis II (prerequisite Probability Specialization)

Sample Surveys and Design of Experiments (prerequisite for Applied Statistics Specialization)

Discrete Mathematics

Categorical Data Analysis

Optimization Techniques

Stochastic Processes

Second Year: Probability specialization

Semester I	Semester II
Statistical Computing I	Weak Convergence and Empirical Processes
Time Series Analysis	Brownian Motion and Diffusions
Martingale Theory	Elective course
Functional Analysis	Elective course
Elective course	Elective course

Second Year: Theoretical Statistics specialization

Semester I	Semester II
Statistical Computing I	Weak Convergence and Empirical Processes
Time Series Analysis	Statistical Inference III
Martingale Theory	Elective course
Statistical Inference II	Elective course
Elective course	Elective course

Second Year: Applied Statistics specialization, Actuarial Statistics track

Semester I	Semester II
Statistical Computing I	Actuarial Models
Time Series Analysis	Survival Analysis
Statistical Inference II	Project
Actuarial Methods	Elective course
Life Contingencies	Elective course

Second Year: Applied Statistics specialization, Biostatistics track

Semester I	Semester II
Statistical Computing I	Clinical Trials
Time Series Analysis	Survival Analysis
Statistical Inference II	Project
Statistical Genomics	Elective course
Elective course	Elective course

Second Year: Applied Statistics specialization, Computational Statistics track

Semester I	Semester II
Statistical Computing I	Statistical Computing II
Time Series Analysis	Inference for High Dimensional Data
Statistical Inference II	Project
Pattern Recognition	Elective course
Elective course	Elective course

Second Year: Applied Statistics specialization, Finance track

Semester I	Semester II
Statistical Computing I	Financial Econometrics
Time Series Analysis	Computational Finance
Statistical Inference II	Project
Quantitative Finance	Elective course
Introductory Economics*/Elective course	Elective course

*Introductory Economics is compulsory for students who would choose the Finance track but did not take a course on Economics at the undergraduate level.

Elective courses in Second Year

Any compulsory course of a specialization/track not chosen by the student (except Introductory Economics)
Dissertation
Advanced Design of Experiments
Advanced Functional Analysis
Advanced Multivariate Analysis
Advanced Nonparametric Inference
Advanced Sample Surveys
Analysis of Directional Data
Asymptotic Theory of Inference
Bayesian Computation
Branching Processes
Commutative Algebra
Descriptive Set Theory
Ergodic Theory
Fourier Analysis
General Topology
Life Testing and Reliability
Markov Processes and Martingale Problems
Mathematical Biology
Ordinary and Partial Differential Equations
Percolation Theory
Random Walks and Electrical Networks
Representation Theory of Finite Groups
Resampling Techniques*
Risk Management
Robust Statistics
Signal and Image Processing
Special Topics in Economics
Special Topics in Finance
Special Topics in Probability
Special Topics in Statistics
Statistical Methods in Demography

Statistical Methods in Epidemiology and Ecology
Stochastic Calculus for Finance**
Theory of Extremes and Point Processes
Theory of Games and Statistical Decisions
Theory of Large Deviations
Theory of Random Graphs

*Available only to those students who have not taken the course in M. Stat 1st Year.
**Available only to those students who have not taken Martingale Theory.

2 Special courses

2.1 Training on "National and International Statistical Systems"

It is a non-credit course offered in between the first and the second semesters of M. Stat 1st year in collaboration with the National Statistical Systems Training Academy (NSSTA) under the Central Statistical Office, New Delhi. The duration of this course is eight days and comprises 40 hours of lectures on Official Statistics. In case of failure in this course, a student may be allowed, in exceptional cases, to undergo training for a second time at his/her expense at the same time in the second year of the M.Stat. programme.

2.2 Project

A student of the second year of M. Stat. opting for the specialization "Applied Statistics" is required do a one semester long project in the second semester under the supervision of a permanent faculty member of the Indian Statistical Institute, provided he/she is not doing a dissertation. The Dean of Studies shall assign a supervisor taking into account the preferences of the student, if any. Students opting for other specializations may also choose to do a one semester year long project as an optional course, provided he/she is not doing a dissertation. In such a case, the student needs to inform the Dean of studies in this regard within the first six weeks of classes in the first semester and the Dean shall assign a supervisor subject to availability of supervisors. Joint supervision of a project by two supervisors is allowed, as long as at least one of them is a member of the faculty of the Institute. The student is required to submit a title and a project proposal to the Dean of Studies before classes of the first semester end.

It is generally expected that a project should be in an area related to Statistics and contain some original contribution by the student on the topic of interest along with real data analyses and/or simulations. The final project report must contain a brief review of the related literature and the new directions explored by the student, if any.

Each project should be reviewed twice by a Committee appointed by the Dean of Studies, which should consist of members as follows:

i. Chairman (a faculty member of ISI, who is not the supervisor of the student);

ii. Convener (the supervisor of the student);

iii. Member (another faculty member of ISI or an external expert).

After the time of the mid-term evaluation, a student is required submit a mid-term report to the Committee members and give a seminar on his/her work. The Committee shall submit its mid- term evaluation report to the Dean of Studies along with a score. The end-term evaluation of the project will be done by the Committee by a deadline to be announced in the Academic Calendar. The student should give a seminar to defend his/her work. A project report must be submitted to the Committee members at least one week before the presentation date. A final evaluation report along must be submitted by the Committee to the Dean of Studies after the seminar. The report must also contain an end-term score corresponding to the end-term presentation as well as a composite score out of 100.

There will be no back paper and/or compensatory examination for the project.

2.3 Dissertation

A student of the second year of M. Stat. may choose to do a yearlong dissertation under the supervision of a permanent faculty member of the Indian Statistical Institute. For students with "Probability" or "Theoretical Statistics" specialization, the dissertation will be treated as a one-semester optional course in the second semester. For students with "Applied Statistics" specialization, the dissertation will be treated as the compulsory Project for the specialization. Under no circumstances, a student will be allowed to do a project and a dissertation as two separate courses in the second year of M. Stat.

In order to be eligible to do a dissertation, a student must

i. obtain at least 85% in aggregate in M. Stat. Ist year and should not have received below 45% marks in any course in the first year of M.Stat.;

ii. find a permanent faculty member of the Institute, who is willing to supervise the dissertation;

iii. submit a title and brief description of the dissertation to the Dean of Studies within the first two weeks of classes of the first semester of the second year of M. Stat.

It is generally expected that a dissertation project should contain original and significant research contribution by the student on a topic related to Statistics or Mathematics. The final dissertation report must contain a brief review of the related literature and the new directions explored by the student.

Each dissertation should be reviewed twice by a Committee appointed by the Dean of Studies, which should consist of members as follows:

i. Chairman (a faculty member of ISI, who is not the supervisor of the student);

ii. Member (a faculty member of ISI or an external expert);

iii. Convener (the supervisor of the student).

A student doing a dissertation is required to submit a mid-term report approved by his/her supervisor to the Committee members and give a seminar on his/her work by a deadline to be announced in the Academic Calendar. The Committee will then submit its mid-term evaluation report to the Dean of Studies along with a score corresponding to the first evaluation. The final evaluation of the dissertation will be done by the Committee after the end of the second semester by a deadline to be announced in the Academic Calendar. The student should give a seminar to defend his/her work. A dissertation report approved and signed by the supervisor must be submitted to the Committee at least two weeks prior to the presentation date. A final evaluation report must be submitted by the Committee to the Dean of Studies after the seminar. The report must also contain an end-term score corresponding to the end-term presentation as well as a composite score out of 100.

There will be no back paper and/or compensatory examination for the dissertation.

3 Detailed Syllabi of the Courses

3.1 First Year Compulsory Courses: B-Stream

Statistical Inference I (for B-stream)

- Game theoretic formulation of a statistical decision problem with illustration. Bayes, minimax and admissible rules. Complete and minimal complete class. Detailed analysis when the parameter space is finite.

- Sufficiency, minimal sufficiency and completeness. Factorization theorem. Convex loss and Rao-Blackwell theorem. Unbiased estimates and Information inequality. Stein estimate and shrinkage for multivariate normal mean. (If time permits: Karlin's theorem on admissibility.)

- Tests of hypotheses. MLR family. UMP and UMP unbiased tests. Detailed analysis in exponential models.
- Discussion of various paradigms of statistical inference. Bayes estimates and tests. Bayesian credible region. Non-informative priors.

References

1. T. S. Ferguson, *Statistical Decision Theory*, Academic Press, 1967
2. E. L. Lehmann and G. Casella, *Theory of Point Estimation*, Second Edition, Springer, 1998
3. E. L. Lehmann and J. P. Romano, *Testing Statistical Hypothesis*, Third Edition, Springer, 2008
4. J. O. Berger, *Statistical Decision Theory*, Springer, 1985

Regression Techniques

- Multiple linear regression; partial and multiple correlations; properties of least squares residuals; forward, backward and stepwise regression; different methods for subset selection.
- Violation of linear model assumptions:
- Lack of fit (linearity): diagnostics and test, Model building.
- Heteroscedasticity: consequences, diagnostics, tests (including Breusch-Pagan test and White's test) and efficient estimation.
- Autocorrelation: consequences, diagnostics, tests (including Durbin-Watson test, Breusch-Godfrey LM test and Durbin's h-test) and efficient estimation.
- Collinearity: consequences, diagnostics and strategies (including ridge and shrinkage regression, LASSO, dimension reduction methods).
- Discordant outlier and influential observations: diagnostics and strategies. Robust regression techniques: LAD, LMS and LTS regression (brief exposure).
- Log-Linear models. Introduction to Generalized Linear Models (GLMs), illustration with logit and probit analysis. Linear predictor, link function, canonical link function, deviance. Maximum likelihood estimation using iteratively re-weighted least square algorithm. Goodness of fit test.

- Introduction to nonparametric regression techniques: Kernel regression, local polynomial, knn and weighted knn methods.
- Data analysis and application of the above methods with computer packages.

References

1. Thomas P. Ryan, *Modern Regression Methods*.
2. Douglas C. Montgomery, *Introduction to Linear Regression Analysis*.
3. David A. Belsley, Edwin Kuh and Roy E. Welsch, *Regression Diagnostics: Identifying Influential Data and Source of Collinearity*.
4. Peter J. Rousseeuw and Annick M. Leroy, *Robust Regression and Outlier Detection*.
5. P. McCullagh and John A. Nelder. *Generalized Linear Models*.
6. Pagan, A. and A. Ullah, *Nonparametric Econometrics*.
7. Matt Wand. and M. C. Jones. *Kernel Smoothing*.
8. J. Fan and I. Gijbels. *Local Polynomial Modelling and its Applications*.

Multivariate Analysis

- Review of: multivariate distributions, multivariate normal distribution and its properties, distributions of linear and quadratic forms, tests for partial and multiple correlation coefficients and regression coefficients and their associated confidence regions.
- Wishart distribution (definition, properties), construction of tests, union-intersection and likelihood ratio principles, inference on mean vector, Hotelling's T2.
- MANOVA.
- Inference on covariance matrices.
- Discriminant analysis.
- Basic introduction to: principal component analysis and factor analysis.
- Practicals on the above topics using statistical packages for data analytic illustrations.

References

1. T. W. Anderson, *An Introduction to Multivariate Statistical Analysis*.

2. R. A. Johnson and D. W. Wichern, *Applied Multivariate Statistical Analysis*.
3. K. V. Mardia, J. T. Kent and J. M. Bibby, *Multivariate Analysis*.
4. M. S. Srivastava and C. G. Khatri, *An Introduction to Multivariate Statistics*.
5. C. R. Rao, *Linear Statistical Inference and its Applications*.

Stochastic Processes

- Poisson process, equivalence of various constructions, basic properties, conditional distribution of arrival times given number of events and its applications, compound Poisson process, inhomogeneous Poisson process.
- Continuous time Markov chains with countable state space, Kolmogorov equations, Birth and Death chains, applications to queuing theory, busy period analysis, network of queues.
- Branching chain, progeny distribution and progeny generating function, extinction probability, geometric growth in the super-critical case, cascade process, applications, multi-type and continuous time branching chains (if time permits).
- Renewal process, renewal theorems, delayed renewal process, Poisson process as a renewal process.
- (If time permits) Some Markov chain models in genetics.

References

1. Sheldon Ross, *Stochastic Processes*.
2. Hoel, Port and Stone, *Stochastic Processes*.
3. S. Karlin, *Stochastic Processes*.
4. Harris, *Branching Processes*.

Categorical Data Analysis

- Visualizing Categorical data. Measures of association. Structural models for discrete data in two or more dimensions.
- Estimation in complete tables. Goodness of fit, choice of a model.
- Generalized Linear Model for discrete data, Poisson and Logistic regression models.

- Log-linear models. Odds-ratio.
- Product multinomials to model sampling from multiple populations. Elements of inference for cross- classification tables. Chi-square approximation for various goodness-of-fit statistics.
- Models for nominal and ordinal response.
- Path models and Structural Equations Modelling.

References

1. Agresti, *An Introduction to Categorical Data Analysis*, Wiley 2007.
2. Bilder and Loughlin, *Analysis of Categorical data with R*, Chapman and Hall/CRC 2014.
3. Kateri, *Contingency Table Analysis*, Springer 2014.
4. Dobson and Barnett, *An Introduction to Generalized Linear Models*, Chapman & Hall/CRC 2008.
5. Hosmer, Lemeshow and Sturdivant, *Applied Logistic Regression*, Wiley 2013.

Large Sample Statistical Methods

- Review of various modes of convergence of random variables and central limit theorems. Continuous mapping theorem. Cramer-Wold device and multivariate central limit theorem. Scheffe's theorem. Polya's theorem. Slutsky's theorem. Law of iterated logarithm (statement only).
- Asymptotic distribution of transformed statistics. Delta method. Derivation of the variance stabilizing formula. Asymptotic distribution of functions of sample moments like sample correlation coefficient, coefficient of variation, measures of skewness and kurtosis.
- Asymptotic distribution of order statistics including extreme order statistics. Asymptotic representation of sample quantiles.
- Large sample properties of maximum likelihood estimates and the method of scoring.
- Large sample properties of parameter estimates in linear models.

- Pearson's chi-square statistic. Chi-square and likelihood ratio test statistics for simple hypotheses related to contingency tables. Heuristic proof for composite hypothesis with contingency tables as examples.

- Large sample nonparametric inference (e. g., asymptotics of U-statistics and related rank based statistics).

- Brief introduction to asymptotic efficiency of estimators.

- (If time permits) Introduction to Edgeworth Expansions.

References

1. R.J. Serfling, *Approximation Theorems in Mathematical Statistics*.

2. C.R. Rao, *Linear Statistical Inference and Its Applications*.

3. A.W. van der Vaart, *Asymptotic Statistics*.

4. E.L. Lehmann, *Elements of Large-Sample Theory*.

5. T.S. Ferguson, *A Course in Large Sample Theory*.

Measure Theoretic Probability

- Motivation: Doing integration beyond Riemann theory, infinite tosses of a fair coin

- Fields, sigma-fields, measures, sigma-finite/finite/probability measures, properties, statement of Caratheodory extension theorem (outline of idea, if time permits). Monotone class theorem, Dynkin's pi-lambda theorem. Radon measures on finite-dimensional Borel sigma-field, distribution functions, correspondence between probability measures on Borel sigma-field and probability distribution functions.

- Measurable functions, basic properties, sigma-fields generated by functions, integration of measurable functions, properties of integrals, MCT, Fatou's Lemma, DCT, Scheffe's theorem. Chebyshev's, Holder's and Minkowski's inequalities. Lp spaces.

- Finite product of measurable spaces, construction of product measures, Fubini's theorem.

- Probability spaces, random variables and random vectors, expected value and its properties. Independence.

- Various modes of convergence and their relation. Uniform integrability (if time permits). The Borel- Cantelli lemmas. Weak Law of large numbers for i.i.d. finite mean case. Kolmogorov 0-1 law,

- Kolmogorov's maximal inequality. Statement of Kolmogorov's three-Series theorem (proof if time permits). Strong law of large numbers for i.i.d. case.

- Characteristic functions and its basic properties, inversion formula, Levy's continuity theorem. Lindeberg CLT, CLT for i.i.d. finite variance case, Lyapunov CLT.

References

1. Robert B. Ash and Catherine A. Doleans-Dade, *Probability and Measure Theory*.

2. Kai Lai Chung, *A Course in Probability Theory*.

3. Patrick Billingsley, *Probability and Measure*.

4. Y.S. Chow and H. Teicher, *Probability Theory*.

5. Rick Durrett, *Probability: Theory and Examples*.

Resampling Techniques

- Introduction: what is resampling? and its purpose. Examples from estimating variance, sampling distribution and other features of a statistic, shortcomings of analytic derivations.

- Different resampling schemes: jackknife, bootstrap, half-sampling.

- Bootstrap in the i.i.d. case: parametric and non-parametric bootstrap, Bayesian bootstrap, consistency and inconsistency of bootstrap, comparison between bootstrap approximation and normal approximation.

- Jackknife in the i.i.d. case: consistency and inconsistency issues, comparison with non-parametric bootstrap.

- Resampling in non-i.i.d. models: need for other resampling schemes, introduction to estimating equation bootstrap and generalized bootstrap.

- Resampling in linear models: special emphasis on residual bootstrap and weighted bootstrap, concept of robust and efficient resampling schemes.

- (If time permits) Discussion on Empirical Likelihood.

References

1. Davidson A.C. and Hinkley D.V. (1997): *Bootstrap Methods and their Applications*.

2. Efron B. and Tibshirani R.J. (1993): *An Introduction to the Bootstrap*.

3. Hall P. (1992): *The Bootstrap and Edgeworth Expansion*.

4. Politis D.N., Romano J.P. and Wolf M. (1999): Subsampling.

5. Shao J. and Tu D. (1995): *The Jackknife and Bootstrap*.

6. Barbe P. and Bertail P (1995): *The Weighted Bootstrap* (Lecture Notes in Statistics, Vol 98).

7. Efron B. (1982): *The Jackknife, the Bootstrap and Other Resampling Plans* (CBMS-NSF Regional Conference Series in Applied Mathematics, No 38).

8. Gine E. (1997): *Lectures on Some Aspects of the Bootstrap* (Lecture Notes in Mathematics, Vol 1665).

9. Mammen E. (1992): *When Does Bootstrap Work? Asymptotic Results and Simulations* (Lecture Notes in Statistics, Vol 77).

3.2 First Year Elective Courses: B-Stream

Metric Topology and Complex Analysis

(Prerequisite for Probability Specialization)

- Metric spaces, open/closed sets, sequences, compactness, completeness, continuous functions and homeomorphisms, connectedness, product spaces, Baire category theorem, completeness of $C[0,1]$ and L^p spaces, Arzela-Ascoli theorem.

- Analytic functions, Cauchy-Riemann equations, polynomials, exponential and trigonometric functions.

- Contour integration, Power series representation of analytic functions, Liouville's theorem, Cauchy integral formula, Cauchy's theorem, Morera's theorem, Cauchy-Goursat theorem.

- Singularities, Laurent Series expansion, Cauchy residue formula, residue calculus.

- Meromorphic functions, Rouche's theorem.

- Fractional linear transformations.

References

1. G.F. Simmons: *Introduction to Topology and Modern Analysis*.

2. J.C. Burkill and H. Burkill: *A Second Course in Mathematical Analysis*.

3. J. Conway: *Functions of One Complex Variable*.

4. L. Ahlfors: *Complex Analysis*.

Abstract Algebra

Same as Algebra II of M.Math.
(Modified as per a decision in the 75th meeting of the Academic Council)

- Review of normal subgroups, quotient, isomorphism theorems, Group actions with examples, class equations and their applications, Sylow's Theorems; groups and symmetry. Direct sum and free Abelian groups. Composition series, exact sequences, direct product and semidirect product with examples. Results on finite groups: permutation groups, simple groups, solvable groups, simplicity of A_n.

- Algebraic and transcendental extensions; algebraic closure; splitting fields and normal extensions; separable, inseparable and purely inseparable extensions; finite fields. Galois extensions and Galois groups, Fundamental theorem of Galois theory, cyclic extensions, solvability by radicals, constructibility of regular n-gons, cyclotomic extensions.

- Time permitting: Topics from Trace and Norms, Hilbert Theorem 90, Artin-Schreier theorem, Transcendental extensions, Real fields.

References

1. J.J. Rotman, An Introduction to the theory of groups, GTM (148), Springer-Verlag (2002).

2. D.S. Dummit and R.M. Foote, Abstract Algebra, John Wiley (Asian reprint 2003).

3. S. Lang, Algebra, GTM (211), Springer (Indian reprint 2004).

4. N.S. Gopalakrishnan, University Algebra, Wiley Eastern (1986).

5. N. Jacobson, Basic Algebra, W.H. Freeman and Co (1985).

6. G. Rotman, Galois theory, Springer (Indian reprint 2005).

7. TIFR pamphlet on Galois theory.

8. Patrick Morandi, Field and Galois Theory, GTM(167) Springer-Verlag (1996).

9. M. Nagata, Field theory, Marcel-Dekker (1977).

Optimization Techniques

- Review of Lagrange method of multipliers, maxima and minima of differentiable functions of several variables, some exercises.

- Convex sets, flats, hyperplanes, interior and closure, compact convex sets, Constrained optimization problems, basic feasible solutions.

- LP fundamentals, Duality, Duality and Primal Dual algorithm. Simplex algorithms.

- Non-linear programming: One-dimensional minimization method, search method, unconstrained and constrained optimization theory and practices.

- Integer Programming Fundamentals, Well-Solved Problems, Cutting Planes Methods, Branch and Bound, Lagrange Relaxations, Strong Valid Inequality.

- Introduction to Bellman's dynamic programming set-up, Bellman's principle of optimality, the use of this principle for solving some problems (such as the knapsack problem, shortest path problem etc.), Vehicle Routing Problem.

References

1. R. Webster, *Convexity*.

2. M.S. Bazaraa and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*.

3. Hamdy A. Taha, *Integer Programming*.

4. Robert Garfinkel and George L. Nemhauser, *Integer Programming*.

5. L. Wolsey, *Integer Programming*.

Sample Surveys and Design of Experiments

(Prerequisite for Applied Statistics Specialization)

Sample Surveys

- Review of equal and unequal probability sampling, Horvitz-Thompson and Yates-Grundy estimators, properties of good estimators by various approaches.
- Unified theory of sampling. Basu's and Godambe's non-existence theorems, exceptions to the latter, uni-cluster sampling design.
- Des Raj's estimator and its symmetrization, Sufficiency in sampling
- Hajek's and Rao's theorems on non-negative MSE estimation. Murthy's strategy, Lahiri-Midzuno- Sen's strategies
- Non-sampling errors. Imputation techniques.
- Randomized Response – Warner's and Simmon's models for attributes.
- Practicals and Simulations.

References

1. W.G. Cochran, *Sampling Techniques*, (1977).
2. M.N. Murthy, *Sampling: Theory and Methods*, (1968).
3. Arijit Chaudhuri, *Modern Survey Sampling*, (2014).
4. Arijit Chaudhuri, *Randomized Response and Indirect Questioning in Surveys*, (2011).

Design of Experiments

- Review of analysis of non-orthogonal block designs under fixed effects models, connectedness, orthogonality and balance; applications.
- BIBD: applications, analysis, construction
- Introduction to row-column designs and their applications.
- Symmetrical factorials, confounding, fractional factorials, introduction to orthogonal arrays and their applications.
- Practicals on the above topics using statistical packages for data analytic illustrations.

References

1. Aloke Dey, *Incomplete Block Designs*.
2. D. Raghavarao and L.V. Padgett, *Block Designs: Analysis, Combinatorics and Applications*.
3. Angela Dean and Daniel Voss, *Design and Analysis of Experiments*.
4. D.C. Montgomery, *Design and Analysis of Experiments*.

3.3 First Year Compulsory Courses: NB-Stream

Statistical Inference I (for NB-stream)

- Review of methods of estimation including method of moments, maximum likelihood and confidence intervals. Sufficiency, factorization theorem, minimal sufficiency, exponential family and completeness. Ancillary statistics and Basu's theorem. Rao-Blackwell theorem. Unbiased estimates and UMVUE. Fisher Information and Cramer-Rao inequality.
- Tests of hypotheses. MP tests, N-P lemma. UMP tests and MLR family. Examples and Illustrations using exponential family models. UMPU tests. Likelihood ratio tests.
- Elements of Bayesian inference including Bayes estimates, credible intervals and tests. Conjugate and Non-informative priors.
- Game theoretic formulation of a statistical decision problem with illustration. Bayes, minimax and admissible rules. Complete and minimal complete class.
- (If time permits) The following topics may be covered: Similarity, Neyman structure, illustrations.

References

1. G. Casella and R. L. Berger, *Statistical Inference*, Second Edition, Duxbury, 2002
2. P. J. Bickel and K. A. Doksum, *Mathematical Statistics*, Second Edition, Volume I, Prentice-Hall, 2001
3. E. L. Lehmann and G. Casella, *Theory of Point Estimation*, Second Edition, Springer, 1998

4. E. L. Lehmann and J. P. Romano, *Testing Statistical Hypothesis*, Third Edition, Springer, 2008

5. T. S. Ferguson, *Statistical Decision Theory*, Academic Press, 1967

6. J. O. Berger, *Statistical Decision Theory*, Springer, 1985

7. L. Wasserman, *All of Statistics* (Part II only), Springer, 2004

Linear Algebra and Linear Models

Linear Algebra

- Review of Vector Space: Subspaces, linear dependence and independence, basis, dimension, sum and intersection of subspaces, inner product and norm, geometric interpretation, Gram-Schmidt orthogonalization, orthogonal projection, projection on a subspace.

- Review of Matrices: Rank, trace, elementary operations, canonical reductions, Kronecker product, orthogonal matrices, symmetric matrices, inverse, sweep-out method, operations with partitioned matrices, determinants.

- Linear equations, homogeneous and inhomogeneous systems, solution space, consistency and general solution, characteristic roots and vectors, Cayley-Hamilton theorem, canonical reduction of symmetric matrices, spectral decomposition, singular values and singular value decomposition.

- Quadratic forms, definiteness, classification and transformations.

References

1. C.R. Rao, *Linear Statistical Inference and its Applications*.

2. A.R. Rao and P. Bhimsankaram, *Linear Algebra*.

3. R.B. Bapat, *Linear Algebra and Linear Models*.

Linear Models

- Linear statistical models, illustrations, Gauss-Markov model, normal equations and least square estimators, estimable linear functions, Best Linear Unbiased Estimators (BLUEs), g-inverse and solution of normal equations, projection operators as idempotent matrices: properties. Error space and estimation space. Variances and co-variances of BLUEs, estimation of error variance.

- Fundamental theorems of least squares and applications to the tests of linear hypotheses, Fisher- Cochran theorem, distribution of quadratic forms. One-way and two-way classification models, ANOVA and ANCOVA. Nested models, Multiple comparisons.
- Introduction to random effects models
- Practicals using statistical packages (such as R).

References

1. C.R. Rao, *Linear Statistical Inference and its Applications*.
2. A.M. Kshirsagar, *A Course in Linear Models*.
3. D.D. Joshi, *Linear Estimation and Design of Experiments*.
4. S.R. Searle, *Linear Models*.
5. F.A. Graybill, *An introduction to Linear Models*, Vol. I.
6. D. Sengupta and S.R. Jammalamadaka, *Linear Models: An Integrated Approach*.

Probability Theory

- Probability distribution functions on real line, extension to a probability measure on a class of subsets of real line, examples followed by the statement on a unique extension. Probability spaces, real random variables, distributions, continuous and discrete random variables. General definition of expected value of a random variable through discrete approximation (outline of the idea without complete proofs).
- Multivariate distributions and properties. Independence. Multivariate densities and multivariate singular distributions. Conditional distributions. Distributions of functions of random vectors and Jacobian formula. Examples of multivariate densities. Multivariate Normal distribution and properties.
- Properties of expectation, linearity, order-preserving property, Holder, Minkowski, Jensen's inequalities, Statements of MCT, Fatou's Lemma and DCT. Expectation of product for independent random variables. Formula for expectation in the discrete and continuous cases (outline only).
- Different modes of convergence and their relations, First and Second Borel-Cantelli Lemmas, Chebyshev inequality, Weak Law of large numbers, Strong Law of large numbers (statement only).

- Characteristic function and its properties. Statements of uniqueness and inversion formula for integrable characteristic functions. Examples (normal, uniform, exponential, double exponential, Cauchy), Levy continuity theorem (statement only), CLT in i.i.d. finite variance case (only sketch of proof).

- Discrete Markov chains with countable state space, Examples including 2-state chain, random walk, birth and death chain, renewal chain, Ehrenfest chain, card shuffling, etc.

- Classification of states, recurrence and transience; absorbing states, irreducibility, decomposition of state space into irreducible classes, Examples.

- Stationary distributions, Ergodic theorem for irreducible recurrent chain (proof only for finite state space), positive and null recurrence. Periodicity, cyclic decomposition of a periodic chain, limit theorem for aperiodic irreducible recurrent chains.

- Poisson process (If time permits): basic properties, conditional distributions of arrival times as order statistics, some applications.

References

1. W. Feller, *Introduction to the Theory of Probability and its Applications*, (Vols. 1 and 2).

2. K.L. Chung, *Elementary Probability Theory*.

3. S.M. Ross, *A First Course in Probability*.

4. R. Ash, *Basic Probability Theory*.

5. P.G. Hoel, S.C. Port and C.J. Stone, *Introduction to Probability Theory*.

6. J. Pitman, *Probability*.

7. P.G. Hoel, S. C. Port and C. J. Stone, *Introduction to Stochastic Processes*.

Analysis I

- Axioms of real number system as complete Archimedean ordered field. Geometric representation of real numbers. Modulus function and Cauchy-Schwarz inequality, Usual distance on Rn, complex numbers as points on R2, Unification as metric spaces, Rational and irrational numbers, examples (with proof) of some irrational numbers, denseness of rational numbers. Cardinality. Uncountability of the set of reals.

- Sequences of real numbers, their lim sup, lim inf and convergence, Convergence of sequences in Rn, Cauchy sequences and completeness of Rn. Some important examples of convergent sequences of real numbers.

- Open and closed sets in R, dense sets in R, Cantor Intersection theorem, Bolzano-Weierstrass theorem and Heine-Borel theorem for R.

- Series of real numbers, convergent, absolutely convergent and conditionally convergent sequence of real numbers, Tests of convergence, rearrangement of series.

- Real valued functions of real numbers, Limits and continuity, Properties of continuous functions.

- Differentiation of real valued functions of real variables, geometric interpretation of derivative, computation of standard derivatives, applications to monotone functions, successive derivatives, Rolle's theorem and Mean value theorem, Taylor series with remainder and infinite Taylor series, L'Hospital Rule, Maxima-Minima, Leibniz Theorem.

- Definite Riemann Integration and its elementary properties, Integrability of functions with finitely many points of discontinuity, Mean-value theorem for Riemann-integration, Fundamental theorem of Calculus, computation of some standard integrals, Integration by parts and change of variable theorem, Interchange of order of integration and limits.

- Sequences of real valued functions of real variables, pointwise and uniformly convergent sequences of functions, Series of functions, Power series and their radii of convergence, examples of trigonometric functions, computation of limits of functions. Weierstrass approximation theorem.

References

1. Stephen Abbott, *Understanding Analysis*.

2. Robert G. Bartle and Donald R. Sherbert, *Introduction to Real Analysis*.

3. Ajit Kumar and S. Kumaresan, *A Basic Course in Real Analysis*.

Regression Techniques

Same as Regression Techniques of B-stream

Introductory Computer Programming

This non-credit course is meant to last one one-semester, but it may be spread over two semesters also.

- Basics in Programming: flow-charts, logic in programming.
- Common syntax (1 week), handling input/output files.
- Sorting.

[These topics should be covered with simultaneous introduction to C/R/Python.]

- Iterative algorithms.
- Simulations from statistical distributions.
- Programming for statistical data analyses: regression, estimation, Parametric tests.

Two lecture hours and two hours of computer class per week. Minimum 50% weight should be for assignments. A final examination is not mandatory for this course.

Large Sample Statistical Methods

Same as Large Sample Statistical Methods of B-stream

Multivariate Analysis

Same as Multivariate Analysis of B-stream

3.4 First Year Elective Courses: NB-Stream

Nonparametric and Sequential Methods

(Prerequisite for Theoretical Statistics Specialization and Applied Statistics Specialization)

Nonparametric Inference

- Formulation of the problem, order statistics and their distributions. Tests and confidence intervals for population quantiles.
- Sign test. Test for symmetry, signed rank test, Wilcoxon-Mann-Whitney test, Kruskal-Wallis test. Estimation of location and scale parameters. Run test, Kolmogorov-Smirnov tests. Measures of association: Kendall's tau, Spearman's rank correlation, Coefficient of concordance-tests of independence.

- Smoothing: Histogram, moving bin, and kernel type density estimation. The curse of dimensionality.

- Concepts of asymptotic efficiency.

References

1. E.L. Lehmann, *Nonparametrics: Statistical Methods Based on Ranks*.

2. Larry, Wasserman, *All of Nonaparametric Statistics*.

3. T.P. Hettmansperger, *Statistical Inference Based on Ranks*.

4. P.J. Bickel and K.A. Doksum, *Mathematical Statistics*.

5. R.L. Berger and G. Casella, *Statistical Inference*.

6. J.D. Gibbons and S. Chakraborti, *Nonparametric Statistical Inference*.

7. M. Hollander, *Nonparametric Statistical Methods* (2014).

Sequential Analysis

- Need for sequential tests. Wald's SPRT, ASN, OC function. Stein's two stage fixed length confidence interval. Illustrations with Binomial and Normal distributions. Elements of sequential estimation.

- (If time permits) (i) empirical likelihood (ii) Introduction to bootstrap and Jackknife methods.

- Practicals using statistical packages.

References

1. David Siegmund, *Sequential Analysis: Tests and Confidence Intervals*.

2. Z. Govindarajulu, *Sequential Statistics*.

3. Nitis Mukhopadhyay and Basil M. de Silva, *Sequential Methods and Their Applications*.

4. Abraham Wald, *Sequential Analysis*.

Measure Theoretic Probability

(Prerequisite for Theoretical Statistics Specialization and Probability Specialization)
Same as Measure Theoretic Probability of B-stream

Analysis II

(Prerequisite Probability Specialization)

- Definition of metric spaces, R, C, R^n, $C[0, 1]$ with uniform metric etc. as examples, open and closed balls, open and closed sets, dense sets and separable metric spaces, sequences in metric spaces and their convergence, Continuity, Cauchy sequences and complete metric spaces, Cantor intersection theorem.
- Compact metric spaces, countable product of metric spaces. Heine-Borel theorem for R^n
- Partial derivatives and directional derivatives, Differentiable functions of several real variables, differentiability of functions with continuous partial derivatives, Jacobians and chain rule, Mean value theorem and Taylor theorem for functions of several real variables, Statements of inverse and implicit function theorems, Lagrange multipliers.
- Proper and improper Riemann integration of functions of several variables, computation of some integrals.
- Holomorphic functions, Cauchy-Riemann equations.
- Power series, Radius of convergence, continuity of power series, termwise derivative of power series, Cauchy product, Exponential and trigonometric functions.
- Path integral, Cauchy theorem and Cauchy integral formula for convex regions.
- Consequences of Cauchy theorem: Taylor series expansion, Maximum modulus principle, Schwarz lemma, Morera's theorem, Zeros, poles and essential singularities of holomorphic functions, residues, Laurent series expansion.
- Contour integral, Residue theorem and computation of some integrals.

References

1. J.C. Burkill and H. Burkill, *A Second Course in Mathematical Analysis*.
2. Walter Rudin, *Principles of Mathematical Analysis*.
3. A.R. Shastri, *Basic Complex Analysis of One Variable*.

Sample Surveys and Design of Experiments

Same as Sample Surveys and Design of Experiments of B-stream

Discrete Mathematics

Combinatorics

- Review of permutation and combinations. Pigeonhole principle and its generalization, Dilworth's Lemma, Introduction to Ramsey theory, Principle of inclusion and exclusion with application to counting derangements.
- Generating functions, definition, operations, applications to counting, integer partitioning, Exponential generating functions, definition, applications to counting permutations, Bell numbers and Stirling number of the second kind.
- Recurrence Relations and its type, linear homogeneous recurrences, inhomogeneous recurrences, divide-and-conquer recurrences, recurrences involving convolution and their use in counting, Fibonacci numbers, derangement, Catalan numbers, Recurrence relation solutions, methods of characteristic root, use of generating functions.

Graph Theory

- Definition of graph and directed graph, definition of degree, subgraph, induced subgraph, paths and walk, connectedness of a graph, connected components.
- Examples of graphs, cycles, trees, forests, integer line and d-dimensional integer lattice, complete graphs, bipartite graphs, graph isomorphism, Eulerian paths and circuits, Hamiltonian paths and circuits.
- Adjacency matrix and number of walks, shortest path in weighted graphs, minimum spanning tree, greedy algorithm and Kruskal algorithms, number of spanning trees, Cayley's theorem, Basics on graph reversal, Breadth-first-Search (BFS) and Depth-first-search (DFS).
- Planarity –definition and examples, Euler's theorem for planar graphs, Dual of a planar graph, Definition of independent sets, colouring, chromatic number of a finite graph, planar graph and chromatic number, five colour theorem for planar graphs, four colour theorem (statement only)
- Flows – definitions and examples, max-flow min-cut theorem.

References

1. J. Matousek and J. Nesetril, *Invitation to Discrete Mathematics*.
2. Fred S. Roberts and B. Tesman, *Applied Combinatorics*.

3. Ronald L. Graham, Donald E. Knuth and O. Patashnika, *Concrete Mathematics*.

4. C.L. Liu, *Elements of Discrete Mathematics*.

5. B. Kolman, R.C. Busby, S.C. Ross and N. Rehman, *Discrete Mathematical Structures*.

6. Martin J. Erickson, *Introduction to Combinatorics*.

7. Frank Harary, *Graph Theory*.

8. Douglas B. West, *Introduction to Graph Theory*.

9. Reinhard Diestel, *Graph Theory*.

Categorical Data Analysis

Same as Categorical Data Analysis of B-stream

Optimization Techniques

Same as Optimization Techniques of B-stream

Stochastic Processes

Same as Stochastic Processes of B-stream

3.5 Second Year Compulsory Courses: Probability specialization

Statistical Computing I

- Review of simulation techniques and their applications.
- Review of re-sampling methods like jackknife, bootstrap and cross-validation.
- Robust measures of multivariate location and scatter: MCD, MVE estimates, Tyler's shape matrix. Independent Component Analysis.
- Introduction to Cluster Analysis, K-means and Hierarchical methods.
- Analysis of incomplete data: EM algorithm, MM (majorization-minimization and minorization- maximization) algorithm.
- Nonparametric regression models: methods based on splines, additive models, projection pursuit models, tree-models, MARS.

- Generalized linear models and generalized additive models.
- Introduction to Markov Chain Monte Carlo techniques with applications, Gibbs sampling, Metropolis-Hastings algorithm.
- Illustrative examples using statistical software.

References

1. S.M. Ross, *Simulation*, Second edition.
2. R.A. Thisted, *Elements of Statistical Computing*.
3. W.N. Venables and B.D. Ripley, *Modern Applied Statistics with S-Plus*, Third Edition.
4. Peter J. Rousseeuw and Annick M. Leroy, *Robust Regression and Outlier Detection*.
5. P. McCullagh and J.A. Nelder, *Generalized Linear Models*.
6. T. Hastie and R. Tibshirani, *Generalized Additive Models*.
7. L. Breiman et al, *Classification and Regression Trees*.
8. Brian Everitt, *Cluster Analysis*.
9. R.J.A. Little, D B. Rubin, *Statistical Analysis with Missing Data*.
10. T. Hastie, R. Tibshirani and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference and Prediction*.

Time Series Analysis

- Exploratory analysis of time series: Graphical display, classical decomposition model, concepts of trend, seasonality and cycle, estimation of trend and seasonal components.
- Stationary time series models: Concepts of weak and strong stationarity, AR, MA and ARMA processes – their properties, conditions for stationarity and invertibility, autocorrelation function (ACF), partial autocorrelation function (PACF), identification based on ACF and PACF, estimation, order selection and diagnostic tests.

- Inference with non-stationary models: ARIMA model, determination of the order of integration, trend stationarity and difference stationary processes, tests of non-stationarity i.e., unit root tests – Dickey-Fuller (DF) test, augmented DF test, and Phillips-Perron test.

- Forecasting: Simple exponential smoothing, Holt-Winters method, minimum MSE forecast, forecast error, in-sample and out-of-sample forecast.

- Modelling seasonal time series: Seasonal ARIMA models, estimation; seasonal unit root test (HEGY test).

- Missing data problem in time series: The average replacement method, and the forecasting replacement method.

- Structural breaks in time series: Quandt-Andrews test for single structural break, estimation of break date, structural break and unit root tests, and Perron's extension of the ADF test.

- Simple state space models: State space representation of ARIMA models, basic structural model, and Kalman recursion.

- Spectral analysis of weakly stationary processes: Spectral density function (s. d. f.) and its properties, s. d. f. of AR, MA and ARMA processes, Fourier transformation and periodogram.

- Adequate data analysis using software packages must be done.

References

1. J.D. Hamilton, *Time Series Analysis*.
2. P.J. Brockwell and R.A. Davis, *Introduction to Time Series Analysis*.
3. C. Chatfield, *Introduction to Time Series*.
4. W.A. Fuller, *Introduction to Statistical Time Series*.
5. T.W. Anderson, *The Statistical Analysis of Time Series*.
6. R.H. Shumway and D.S. Stoffer, *Time Series Analysis and Its Applications*.
7. G S. Maddala and In-Moo Kim, *Unit Roots, Cointegration and Structural Change*.
8. R.S. Tsay, *Analysis of Financial Time Series*.

Martingale Theory

- Absolute continuity and singularity of measures. Hahn-Jordon decomposition, Radon-Nikodym Theorem, Lebesgue decomposition.

- Conditional expectation -Definition and Properties. Regular conditional probability, proper RCP. Regular conditional distribution.

- Discrete parameter martingales, sub-and super-martingales. Doob's Maximal Inequality, Upcrossing inequality, martingale convergence theorem, Lp inequality, uniformly integrable martingales, reverse martingales, Levy's upward and downward theorems. Stopping times, Doob's optional sampling theorem. Discrete martingale transform, Doob's Decomposition Theorem.

- Applications of martingale theory: SLLN for i.i.d. random variables. Infinite products of probability spaces, Hewitt-Savage 0-1 Law. Finite and infinite exchangeable sequence of random variables, de Finetti's Theorem. SLLN for U-Statistics for exchangeable data.

- Introduction to continuous parameter martingales: definition, examples and basic properties.

- (If time permits) Martingale Central Limit Theorem and applications, Azuma-Hoeffding Inequality and some applications.

References

1. Y.S. Chow and H. Teicher, *Probability Theory*.

2. Leo Breiman, *Probability Theory*.

3. Jacques Nevue, *Discrete Parameter Martingales*.

4. P. Hall and C.C. Heyde, *Martingale Limit Theory and its Application*.

5. R. Durret, *Probability Theory and Examples*.

6. P. Billingsley, *Probability and Measure*.

Functional Analysis

- Basic metric spaces and locally compact Hausdorff spaces.

- Normed linear spaces, Banach spaces. Bounded linear operators. Dual of a normed linear space.
- Hahn-Banach theorem, uniform boundedness principle, open mapping theorem, closed graph theorem.
- Computing the dual of well-known Banach spaces. L_p spaces. Riesz representation theorem and Stone-Weierstrass theorem.
- Hilbert spaces, adjoint operators, self-adjoint and normal operators, spectrum, spectral radius, analysis of the spectrum of a compact operator on a Banach space, spectral theorem for bounded self-adjoint operators.
- (If time permits) Spectral theorem for normal and unitary operators.

References

1. W. Rudin, *Real and Complex Analysis*, McGraw-Hill (1987).
2. W. Rudin, *Functional analysis*, McGraw-Hill (1991).
3. J.B. Conway, *A Course in Functional Analysis*, GTM (96), Springer-Verlag (1990).
4. K. Yosida, *Functional analysis*, Grundlehren der Mathematischen Wissenschaften (123), Springer- Verlag (1980).

Weak Convergence and Empirical Processes

- Probability measures on metric spaces. Weak convergence of probability measures on metric spaces. Portmanteau theorem. Convergence determining classes. Continuity theorem. Prohorov's theorem. Levy-Prohorov metric, Skorohod representation theorem.
- Weak convergence on $C(0,1)$, Arzela-Ascoli theorem, sufficient conditions for weak convergence on $C(0,1)$.
- Construction of Weiner measure on $C(0,1)$, Donsker's theorem, Application of continuity theorem to derive distributions of certain functionals of BM. Kolmogorov-Smirnov statistics. Weiner measure on $C(0,\infty)$.
- $D(0,1)$, Skorohod topology on $D(0,1)$, compactness on $D(0,1)$. Weak convergence of probability measures on $D(0,1)$. Empirical distribution functions, Donsker's Theorem on $D(0,1)$.

- Vapnik-Chervonenkis Theory in Empirical processes: Glivenko-Cantelli classes, Donsker classes, Vapnik-Chervonenkis classes, Shattering and VC-index, VC inequality with applications to convergence results.

References

1. P. Billingsley, *Weak Convergence of Probability Measures*.
2. K.R. Parthasarathy, *Probability measures on Metric Spaces*.
3. D. Pollard, *Empirical Processes*.

Brownian Motion and Diffusions

- Introduction to Brownian Motion, Kolmogorov Consistency theorem, Kolmogorov Continuity theorem, Construction of BM. Basic Martingale Properties and path properties -including Holder continuity and non-differentiability. Quadratic variation.
- Markov Property and strong Markov property of BM, reflection principle, Blumenthal's 0-1 law. Distributions of first passage time and of running maximum of BM.
- Brownian Bridge as BM conditioned to return to zero.
- Ito Integral with respect to BM, properties of Ito integral. Ito formula, Levy characterization, representation of continuous martingales of Brownian filtration.
- Continuous path Polish space-valued markov processes, Feller processes, Associated semigroup operators, resolvent operators and generators on the Banach space of bounded continuous functions. Generator of BM.
- Ito diffusions, Markov property of Ito diffusions, Generators of Ito diffusions.

References

1. K. Ito, *TIFR Lecture Notes on Stochastic Processes*.
2. I. Karatzas and S.E. Shreve, *Brownian Motion and Stochastic Calculus*.
3. D. Freedman, *Brownian Motion and Diffusion*.
4. H.P. Mckean, *Stochastic Integrals*.

3.6 Second Year Compulsory Courses: Theoretical Statistics specialization

Statistical Computing I

Same as Statistical Computing of Probability specialization

Time Series Analysis

Same as Time Series Analysis of Probability specialization

Martingale Theory

Same as Martingale Theory of Probability specialization

Statistical Inference II

- Introduction to the Bayesian paradigm. Review of inference based on posterior distribution –point estimation and credible sets. Predictive distributions. Illustration with examples of one-parameter and multiparameter models using conjugate and noninformative priors.
- Large sample properties –Consistency and asymptotic normality of posterior distribution, Laplace's method.
- Bayesian testing and Model selection. BIC, DIC. Objective Bayes factors. Intrinsic priors. Bayesian variable selection.
- Comparison of p-value and posterior probability of H0 as measures of evidence. Bayesian p-value.
- Brief discussion on Bayesian computation.
- Bayesian approaches to some common problems in Inference including Linear Regression.
- Application of Stein estimation, parametric empirical Bayes and hierarchical Bayes approaches to high-dimensional problems including multiple tests.

References

1. A. Gelman, J.B. Carlin, H.S. Stern, D.B. Dunson, A. Vehtari and D.B. Rubin, *Bayesian Data Analysis* (3rd Edition).

2. B.P. Carlin and T.A. Louis, *Bayesian Methods for Data Analysis* (3rd Edition).

3. J.K. Ghosh, M. Delampady and T. Samanta, *An Introduction to Bayesian Analysis: Theory and Methods*.

4. J.O. Berger, *Statistical Decision Theory and Bayesian Analysis* (2nd Edition).

5. C.P. Robert, *The Bayesian Choice* (2nd Edition).

6. P. Congdon, *Bayesian Statistical Modelling* (2nd Edition).

Weak Convergence and Empirical Processes

Same as Weak Convergence and Empirical Processes of Probability specialization

Statistical Inference III

- Overview of classical inference.
- Principles of data reduction:
  a) Sufficiency: Proof of Factorization Theorem for the Dominated case. Examples and applications with emphasis on Exponential families.
  b) Invariance: Invariant decision rules, equivariant estimation, invariant tests; discussion on admissibility, minimax property etc. of invariant rules. Relation between Sufficiency and Invariance.
  c) Partial Likelihood (with illustrations).
- Foundations of statistics: Coherence, Likelihood principle and justification for the conditional Bayesian approach.
- Multiple hypothesis testing: Concepts of familywise error rate(FWER) and False Discovery Rate (FDR). Procedures for controlling FWER and FDR. False discovery rate control under dependence.

References

1. E.L. Lehmann and J.P. Romano, *Testing Statistical Hypotheses*.

2. E.L. Lehmann, *Theory of Point Estimation*.

3. M.J. Schervish, *Theory of Statistics*.

4. J.K. Ghosh (Ed.), *Statistical Information and Likelihood: A collection of critical essays by Dr. D. Basu*.

5. B. Efron, *Large-scale Inference: Empirical Bayes Methods for Estimation, Testing and Prediction*.

6. Colin Blyth, “Subjective vs. Objective Methods of Statistics”, *The American Statistician*, Vol 26, No 3 (Jun, 1972), pp. 20-22.

7. W.J. Hall, R.A. Wijsman and J.K. Ghosh, “The relationship between sufficiency and invariance with applications in sequential analysis”, *The Annals of Mathematical Statistics*, Vol 36, Vol 2 (Apr. 1965), pp. 575-614.

8. Y. Benjamini and Y. Hochberg, “Controlling the false discovery rate: A practical and powerful approach to multiple testing”, *Journal of the Royal Statistical Society. Series B (Methodological)*, Vol. 57, No. 1 (1995), pp. 289-300.

9. J.D. Storey, “A direct approach to false discovery rates”, *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, Vol. 64, No. 3 (2002), pp. 479-498.

10. J.D. Storey, J.E. Taylor and D. Siegmund, “Strong control, conservative point estimation and simultaneous conservative consistency of false discovery rates: a unified approach”, *Journal of the Royal Statistical Society*, Series B (Statistical Methodology), Vol. 66, No. 1 (2004), pp. 187- 205.

11. Y. Benjamini and D. Yekutieli, “The control of false discovery rate in multiple testing under dependency”, *The Annals of Statistics*, Vol. 29, No. 4 (2001), pp. 1165-1188.

12. S.K. Sarkar, “Some results on false discovery rate in stepwise multiple testing procedures”, *The Annals of Statistics*, Vol. 30, No. 1 (2002), pp. 239-257.

3.7 Second Year Compulsory Courses: Applied Statistics specialization

3.7.1 Actuarial Statistics track

Statistical Computing I

Same as Statistical Computing I of Probability specialization

Time Series Analysis

Same as Time Series Analysis of Probability specialization

Statistical Inference II

Same as Statistical Inference II of Theoretical Statistics specialization

Actuarial Methods

(Modified as per a decision in the 74th meeting of the Academic Council)

- Review of decision theory and actuarial applications.
- Loss distributions: Modelling of individual and aggregate losses, moments, deductibles and retention limits, proportional and excess-of-loss reinsurance, share of claim amounts, parametric estimation with incomplete information, fitting distributions to claims data, goodness-of-fit measures.
- Risk models: Models for claim number and claim amount in short-term contracts, assumptions, moments, compound distributions, moments of insurer's and reinsurer's share of aggregate claims.
- Copulas: Basic concept, dependence or concordance, upper and lower tail dependence, selection of parametric models, Archimedean family of copulas.
- Extreme value theory: Extreme value distributions and their properties, tail weight measures and interpretation.
- Time series analysis: Review of stationarity and ARMA/ARIMA models, filters, random walks, multivariate AR model, cointegrated time series, multivariate Markov models, model identification, diagnostics, non-stationary/non-linear models, application to economic variables, forecasting.
- Machine learning principles: bias/variance trade-off, cross-validation, regularisation, black-box software, performance evaluation (precision, recall, F1 score, ROC curve, confusion matrix etc), PCA, K-means clustering).

References

1. N.L. Bowers, H.U. Gerber, J.C. Hickman, D.A. Jones and C.J. Nesbitt, *Actuarial Mathematics*, 2nd ed. Society of Actuaries, 1997.
2. S.A. Klugman, H.H. Panjer, G.E. Willmotand and G.G. Venter, *Loss Models: From Data to Decisions*. 3rd Edition, John Wiley and Sons, 2008.
3. C.D. Daykin, T. Pentikainen and M. Pesonen, *Practical Risk Theory for Actuaries*. Chapman and Hall, 1994.

4. S.D. Promishow, *Fundamentals of Actuarial Mathematics*, John Wiley, 2011.

5. P.J. Boland, *Statistical and Probabilistic Methods in Actuarial Science*, Chapman and Hall, 2007.

6. K.P. Murphy, *Machine Learning: A Probabilistic Perspective*, MIT Press, 2012.

7. T. Hastie, R. Tibshirani and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference and Prediction*, Springer, 2001.

Life Contingencies

(Modified as per a decision in the 74th meeting of the Academic Council)

- Data and basics of modelling: Principles of actuarial modelling, use of general cash flow models for financial transactions.

- Interest rates: Rates at different time periods, real and money interest rates, time value of money, present value and accumulated value of cash flows under different scenarios, notations and derivations of various compound interest functions, term structure of interest rates, duration, convexity and immunisation of cash flows.

- Equation of value: Definition, applications, use in project appraisals.

- Revision of life tables: notations, probability expressions, approximations, select and ultimate tables.

- Assurance and annuity contracts: definitions of benefits and premiums, various types of assurances and annuities, present value, formulae for mean and variance of various continuous and discrete payments, various conditional probabilities from ultimate and select life tables, related actuarial symbols, inter-relations of various types of payments, contracts with profits through reversionary/terminal bonus, unit-linked contracts, accumulating with-profit contracts with benefits in the form of accumulating fund of premiums.

- Functions of two lives: cash-flows contingent on death/survival of either or both of two lives, functions dependent on a fixed term and on age.

- Cash-flow models for multiple decrements: Markov model, expected present values of cash flows, projecting and valuing expected cash flows.

- Calculation of various payments from life tables: principle of equivalence, net premiums, prospective and retrospective provisions/reserves.

- Gross premiums and reserves: Various expenses, role of inflation, calculation of gross premium with future loss and equivalence principle for various types of contracts, alternative principles, calculation of gross premium provisions/reserves, gross premium retrospective provisions, recursive relations.
- Death strain at risk: Actual and expected death strain, mortality profit/loss.
- Projected expected future cash flows: profit tests and profit vector, profit signature, net present value and profit margin, use of profit test in product pricing and determining provisions, eliminating future negative cash flow through non-unit reserves for unit-linked contracts.

References

1. N.L. Bowers, H.U. Gerber, J.C. Hickman, D.A. Jones, and C.J. Nesbitt, *Actuarial Mathematics*, 2nd ed. Society of Actuaries, 1997.
2. A. Neill, *Life Contingencies*. Heinemann, 1977.
3. B. Benjamin and J.H. Pollard, *The Analysis of Mortality and Other Actuarial Statistics*, 3rd ed. Institute of Actuaries and Faculty of Actuaries, 1993.
4. P.M. Booth, R.G. Chadburn, D.R. Cooper, S. Haberman and D.E. James, *Modern Actuarial Theory and Practice*, Chapman and Hall, 1999.
5. S.D. Promishow, *Fundamentals of Actuarial Mathematics*, John Wiley, 2011.

Actuarial Models

(Modified as per a decision in the 74th meeting of the Academic Council)

- Review of stochastic processes: Various types of stochastic processes, including counting processes; their actuarial applications.
- Review of Markov chain: time inhomogeneous chain; frequency based experience rating and other applications; simulation.
- Markov jump process: Poisson process, Kolmogorov equations, illness-death and other survival models, effect of duration of stay on transition intensity, simulation.
- Estimation of transition intensities: maximum likelihood estimators, asymptotic distribution, Poisson approximation, stratification for heterogeneous data, principle of correspondence.

- Central Exposed to Risk: data type, computation, estimation of transition probabilities, census approximation of waiting times, rate intervals, census formulae for various definitions of age.

- Graduated estimates: reasons for comparison of crude estimates of transition intensities/probabilities to standard tables, statistical tests and their interpretations, test for smoothness of graduated estimates, graduation through parametric formulae, standard tables and graphical process, modification of tests for comparing crude and graduated estimates and to allow for duplicate policies.

- Mortality projection: Approaches based on extrapolation, explanation and expectation, Lee–Carter, age-period-cohort and p-spline regression models, forecast error.

- Survival models and methods: Future life random variable and related actuarial notations, two-state model for single decrement, review of parametric and nonparametric methods of inference for censored data, asymptotic inference under Cox model.

References

1. N.L. Bowers, H.U. Gerber, J.C. Hickman, D.A. Jones, and C.J. Nesbitt, *Actuarial Mathematics*, 2nd ed. Society of Actuaries, 1997.

2. V.G. Kulkarni, *Modelling, Analysis, Design, and Control of Stochastic Systems*. Springer, 1999.

3. G. Grimmett and D. Stirzaker, *Probability and Random Processes*, 3rd ed. Oxford University Press, Oxford, 2001.

4. E. Marubini and M.G. Valsecchi, *Analysing Survival Data from Clinical Trials and Observational Studies*, John Wiley and Sons, 1995.

Survival Analysis

- Introduction: Type of data (uncensored, censored, grouped, truncated); Dependence on covariates; Different end points.

- Failure time models: Exponential, Weibull and Gamma. Discrete hazard.

- Likelihood based inference for censored data: Construction of likelihood for different types of censoring; Maximum likelihood estimation (Newton-Raphson method, EM algorithm); Asymptotic likelihood theory (statement of results only); Testing of hypotheses in parametric models.

- Nonparametric inference: Life Table estimates;
- Kaplan-Meier estimate; Nelson-Aalen estimate; Two- sample problem.
- Regression models: Exponential and Weibull regression; Proportional Hazard and Accelerated Life Time models; Discrete regression models; Two-sample problem using regression models.
- Proportional Hazard model: Marginal and Partial likelihoods; Estimation of baseline survival function; Inclusion of strata; Time dependent covariates; Scope and validity of the PH model.
- Accelerated Life Time model: Maximum likelihood estimation; Least square estimation; Linear rank test.
- Competing risks: Cause specific hazard/Multiple decrement model; PH model for competing risks data; Multiple failure time data.
- Counting Process Theory; Multiplicative intensity model; Martingale theory and stochastic integrals; Nelson-Aalen Estimator.

References

1. J.D. Kalbfleisch and R.L. Prentice, *The Statistical Analysis of Failure Time Data*.
2. R.G. Miller, *Survival Analysis*.
3. D.R. Cox and D. Oakes, *Analysis of Survival Data*.
4. Jerald F. Lawless, *Statistical Models and Methods for Lifetime Data*.
5. John P. Klein and Melvin L. Moeschberger, *Survival Analysis: Techniques for Censored and Truncated Data*.

3.7.2 Biostatistics track

Statistical Computing I

Same as Statistical Computing I of Probability specialization

Time Series Analysis

Same as Time Series Analysis of Probability specialization

Statistical Inference II

Same as Statistical Inference II of Theoretical Statistics specialization

Statistical Genomics

- Review of Hardy-Weinberg Equilibrium and allele frequency estimation.
- Construction of Pedigree Likelihoods.
- Linkage and recombination.
- Parametric and ASP methods for detecting linkage for binary traits.
- Sib-pair methods for Detecting Linkage for Quantitative Traits: Haseman-Elston approach.
- Linkage Disequilibrium.
- Review of genetic case-control studies.
- Population Stratification Issues for genetic case-control studies.
- Association tests based on family-data: TDT, Sib-TDT, PDT.
- Statistical issues in Genome-wide association studies: Data quality checks, Imputation, Multiple testing.
- Evolution of DNA sequences: Kimura's two parameter and Jukes Cantor model.
- Pairwise Sequence Alignment Algorithms: Needleman Wunsch and Smith-Waterman.
- Basic Local Alignment Search Tool.
- Construction of evolutionary trees using UPGMA and Neighbour Joining.

References

1. A Statistical Approach to Genetic Epidemiology: Concepts and Applications; Andreas Ziegler, Inke Konig, Friedrich Pahlke
2. Statistics in Human Genetics: Pak C Sham
3. Thomas D. Duncan, *Statistical Methods in Genetic Epidemiology*.
4. David J. Balding, M. Bishop, Chris Cannings (Eds.), *Handbook of Statistical Genetics*.

5. Richard Durbin, Sean Eddy, Anders Krogh and Graeme Mitchison, *Biological Sequence Analysis: Probabilistic Models of Proteins and Nucleic Acids*.

Clinical Trials

- Introduction, ethical issues, protocols, comparative and controlled trials. Different phases.
- Randomization. Different types of biases.
- Sample size determination.
- Phase I trial, dose response studies. Phase II trial.
- Phase III trial, sequential allocation.
- Group sequential design, type I error spending function.
- Treatment adaptive allocation.
- Response-adaptive allocation, play-the-winner, randomized play-the-winner, design for continuous responses, optimal designs.
- Delayed responses, Longitudinal responses, Crossover designs, covariates and surrogate responses.
- Analysis of data: generalized linear model, quasilikelihood and generalized estimating equations.
- Bayesian designs and analysis.
- Some real clinical trial example and illustration.

References

1. J.N.S. Mathews, *An Introduction to Randomized Controlled Clinical Trials*, 2nd Ed.
2. A.C. Atkinson and A. Biswas, *Randomised Response-Adaptive Designs in Clinical Trials*.
3. John Whitehead, *The Design and Analysis of Sequential Clinical Trials*.
4. Stuart J. Pocock, *Clinical Trials: Practical Approach*.

Survival Analysis

Same as Survival Analysis of Actuarial Statistics track

3.7.3 Computational Statistics track

Statistical Computing I

Same as Statistical Computing I of Probability specialization

Time Series Analysis

Same as Time Series Analysis of Probability specialization

Statistical Inference II

Same as Statistical Inference II of Theoretical Statistics specialization

Pattern Recognition

- Introduction to supervised and unsupervised pattern classification.
- Supervised classification
  - Loss and risk function in classification, Admissible rules, Bayes and minimax rules.
  - Fisher's linear discriminant function, linear and quadratic discriminant analysis, regularized discriminant analysis, logistic regression.
  - Other linear classifiers: SVM and Distance Weighted Discrimination, Nonlinear SVM. Kernel density estimation and kernel discriminant analysis.
  - Nearest neighbour classification.
  - Classification under a regression framework: additive models, projection pursuit, neural network, classification using kernel regression.
  - Classification tree and random forests.
- Unsupervised classification
  - Hierarchical and non-hierarchical methods: k-means, k-medoids and linkage methods
  - Cluster validation indices: Dunn index, Gap statistics.

- Clustering using Gaussian mixtures.

- Computer applications using R and other packages.

References

1. Duda, Hart, Stork, *Pattern Classification*.
2. Hastie, Tibshirani, Friedman, *Elements of Statistical Learning*.
3. Ripley, *Pattern Recognition and Neural Networks*.
4. Wand and Jones, *Kernel Smoothing*.
5. Vapnik, *Elements of Statistical Learning*.
6. Bugres, *Tutorial on SVM*.
7. Breiman, Friedman, Olsen, Stone: CART

Statistical Computing II

- Kernels and local polynomials.
- Local likelihood methods.
- Wavelet smoothing.
- Genetic algorithm and Simulated annealing.
- Bump Hunting algorithms.
- Multidimensional scaling and Self-organizing maps.
- Graphical models.
- RKHS and associate statistical methods.
- Ensemble methods: Bagging, boosting, stacking, random forests.
- Introduction to high dimension, small sample size problems.

References

1. M.P. Wand and M.C. Jones (1995). *Kernel Smoothing*.
2. J. Fan and I. Gijbels (1996). *Local Polynomial Modelling and Its Applications*.

3. T. Hastie, R. Tibshirani and J. Friedman (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*.
4. B. Vidakovic (1999). *Statistical Modelling by Wavelets*.
5. G. Kaiser (2010). *A Friendly Guide to Wavelets*.
6. R.L. Haupt and S.E. Haupt (2004). *Practical Genetic Algorithms*.
7. P.J.M. van Laarhoven and E.H.L. Aarts (1992). *Simulated Annealing: Theory and Applications*.
8. R.O. Duda, P.E. Hart and D.G. Stork (2001) *Pattern Classification*.
9. D. Edwards (2000). *Introduction to Graphical Modelling*.
10. J. Whittaker (2009). *Graphical Models in Applied Multivariate Statistics*.
11. S. Højsgaard, D. Edwards and S. Lauritzen (2012). *Graphical Models with R*.
12. G. Wahba (1990). *Spline Models for Observational Data*.
13. B. Schölkopf and A.J. Smola (2002) *Learning with Kernels: Support Vector Machines*.
14. L. Rokach (2009) *Pattern Classification using Ensemble Methods*.
15. C. Zhang and Y. Yunqian Ma (2012). *Ensemble Machine Learning: Methods and Applications.*

Inference for High Dimensional Data

- Motivating examples of high dimensional data (large p, small n) and the need to move beyond classical estimation methods. Examples from genomics, machine learning, economics/finance or any other field.
- Variable selection methods: best subset selection, stepwise forward, Stepwise backward, AIC, BIC, C_p.
- Introduction to the LASSO. Discussion and proofs for various properties of the LASSO, including its variable selection properties (for p¿n), asymptotic distribution (in fixed p set up). The LARS method by Efron (2004); the Adaptive Lasso and its theoretical properties. Specifically focussing on the case where $p > n$, where the choice of the initial estimator plays an important role.

- Multiple testing in high dimensional set up, p-values in high dimensional set up (based on works by Buhlmann).
- Basics about high dimensional covariance matrix estimation. Different approaches.
- Testing for high dimensional data (basic two-sample test).
- High dimensional classification and clustering.

References

1. P. Buhlmann and S. van de Geer, *Statistics for High-dimensional Data*.
2. C. Giraud, *Introduction to High-Dimensional Statistics*.
3. I. Koch, *Analysis of Multivariate and High-dimensional Data*.
4. T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning* (Second ed.).
5. M. Pourahmadi, *High-dimensional Covariance Estimation*.
6. J. Fan and J. Lv (2010), A selective overview of variable selection in high dimensional feature space. *Statist Sinica* 20 (1), 101–148.
7. K. Knight and W. Fu (2000), Asymptotics for lasso-type estimators. *Ann. Statist.* 28 (5), 1356– 1378.
8. L. Wasserman and K. Roeder (2009), High-dimensional variable selection. *Ann. Statist.* 37 (5A), 2178–2201.
9. Z. Bai and H. Saranadasa (1996), Effect of high dimension: by an example of a two sample problem. *Statist. Sinica* 6 (2), 311–329.
10. T.T. Cai, W. Liu, and Y. Xia (2014), Two-sample test of high dimensional means under dependence. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* 76 (2), 349–372.

3.7.4 Finance track

Statistical Computing I

Same as Statistical Computing I of Probability specialization

Time Series Analysis

Same as Time Series Analysis of Probability specialization

Statistical Inference II

Same as Statistical Inference II of Theoretical Statistics specialization

Quantitative Finance

- Corporate Finance: Discount Factors, Betas, Mean-Variance Frontiers, Efficient Portfolios, CAPM.
- Fixed Income Securities: Treasury bills and bonds, STRIPS, defaultable bonds, mortgage-backed securities like Collateraized Mortgage Obligations and derivative securities like swaps, caps, floors, and swaptions; relation between yields and forward rates, and factor models of yield curve dynamics.
- Asset Pricing: Arbitrage, complete markets, preliminaries of Martingales, risk-neutral measure, Fundamental Theorems.
- Brownian motion, Stochastic Integration and Ito's formula, Black Scholes option pricing and hedging, Cameron Martin Formula and Barrier Options, and Girsanov's Theorem.
- Risk Management including VaR, expected shortfall, coherent risk measures, and the Basel accords.

References

1. J. Berk and P. DeMarzo, *Corporate Finance*.
2. B. Tuckman and A. Serrat, *Fixed Income Securities*.
3. D. Duffie, *Dynamic Asset Pricing Theory*.
4. J.H. Cochrane, *Asset Pricing*.
5. J.C. Hull, *Options, Futures and Other Derivatives*.
6. S.E. Shreve, *Stochastic Calculus for Finance II*.
7. D. Duffie and K.J. Singleton, *Credit Risk: Pricing, Measurement and Management*.

Introductory Economics

This course is compulsory for students without having taken a course on Economics at the undergraduate level; it cannot be chosen by anyone as an elective course.

Macroeconomics

- National income accounting: different concepts and three methods of measurement, circular flow of income.
- Determination of equilibrium income (employment): Classical model, simple Keynesian model and its extensions to government sector and open economy.
- Money market: Supply of money and monetary policy, Demand for money. Determination of equilibrium income and interest: IS-LM model in a closed economy. Determination of equilibrium price: Aggregate supply-demand model.

Microeconomics

- Theory of consumer behaviour: preference orderings, demand, duality theory, revealed preference, aggregate demand.
- Theory of firm: production sets, cost minimization, profit maximization, supply, duality theory, aggregate supply.
- Equilibrium in a single market: stability and comparative statics.
- Imperfect competition and market structure.

Development Economics

- Developed vs. underdeveloped economy; features of backward agriculture; dual economy and problems of industrialization; problem of unemployment; poverty and inequality.

References

1. R. Dornbusch and S. Fischer, *Macroeconomics*.
2. W.H. Branson, *Macroeconomic Theory*.
3. N. Gregory Mankiw, *Macroeconomics*.
4. M.P. Todaro, *Economic Development*.

5. P.A. Samuelson and H.D. Nordhaus, *Economics*.
6. A. Mas-Colell, M. Whinston and J. Green, *Microeconomic Theory*.
7. H. Varian, *Microeconomic Analysis*.

Financial Econometrics

- Analysis of Panel Data: Fixed effects model, random effects model (error components model), fixed or random effects. Wu-Hausman test, and dynamic panel model.
- Generalized Method of Moments (GMM): Orthogonality conditions, and properties of the GMM estimator.
- Simultaneous Equations System: Structural and reduced forms, least squares bias problem; identification problem, and estimation methods.
- Cointegration: Concept, two variable model, Engle-Granger method; vector autoregression (VAR), system estimation method – Johansen procedure, vector error correction model(VECM), and tests for cointegration-trace test and max eigenvalue test, and Granger causality. v. ARCH and SV Models: Properties of ARCH/GARCH/SV models, different interpretations, some important generalizations like the EGARCH and GJR models, estimation and testing, and ARCH-M model.

References

1. William H. Greene, *Econometric Analysis*.
2. J. Campbell, A. Lo and C. Mackinlay, *The Econometrics of Financial Markets*.
3. P.J. Brockwell and R.A. Davis, *Introduction to Time Series and Forecasting*.
4. C. Brooks, *Introductory Econometrics for Finance*.
5. H. Lutkepohl and M. Kratzig, *Applied Time Series Econometrics*.
6. G.S. Maddala and In-Moo Kim, *Unit Roots, Cointegration and Structural Break.*

Computational Finance

- Numerical methods relevant to integration, differentiation and solving the partial differential equations of mathematical finance: examples of exact solutions including Black Scholes and its relatives, finite difference methods including algorithms and question of stability and convergence, treatment of near and far boundary conditions, the connection with binomial models, interest rate models, early exercise, and the corresponding free boundary problems, and a brief introduction to numerical methods for solving multi-factor models.

- Simulation including random variable generation, variance reduction methods and statistical analysis of simulation output. Pseudo random numbers, Linear congruential generator, Mersenne twister RNG. The use of Monte Carlo simulation in solving applied problems on derivative pricing discussed in the current finance literature. The technical topics addressed include importance sampling, Monte Carlo integration, Simulation of Random walk and approximations to diffusion processes, martingale control variables, stratification, and the estimation of the "Greeks. " Application areas include the pricing of American options, pricing interest rate dependent claims, and credit risk. The use of importance sampling for Monte Carlo simulation of VaR for portfolios of options.

- Statistical Analysis of Financial Returns: Fat-tailed and skewed distributions, outliers, stylized facts of volatility, implied volatility surface, and volatility estimation using high frequency data.

- If time permits: Copulas, Hedging in incomplete markets, American Options, Exotic options, Electronic trading, Jump Diffusion Processes, High-dimensional covariance matrices, Extreme value theory, Statistical Arbitrage.

References

1. P. Glasserman, *Monte Carlo Methods in Financial Engineering*.

2. D. Ruppert, *Statistics and Data Analysis for Financial Engineering*.

3. R. Carmona, *Statistical Analysis of Financial Data in S-Plus*.

4. N.H. Chan, *Time Series: Applications to Finance*.

5. R.S. Tsay, *Analysis of Financial Time Series*.

6. J. Franke, W.K. Härdle and C.M. Hafner, *Statistics of Financial Markets: An Introduction*.

3.8 Second Year Elective Courses

(In alphabetical order)

Advanced Design of Experiments

Prerequisite: Sample Surveys and Design of Experiments

- Optimality criteria, A-, D-, E-optimality, universal optimality of BBD and generalized Youden Square Designs.
- Hadamard matrices and Orthogonal arrays, constructions, Rao's bound
- Orthogonal arrays as fractional factorial plans, main effect plans for 2-level factorials.
- Response surface designs, method of steepest ascent, canonical analysis and ridge analysis of fitted surface.
- A selection of topics from the following:
- Asymmetric factorials, orthogonal factorial structure, Kronecker calculus for factorials, construction.
- Cross-over designs, applications, analysis and optimality.
- PBIB designs with emphasis on group divisible designs.
- Robust designs and Taguchi methods; Mixture experiments; Nested designs; Optimal regression designs for multiple linear regression and quadratic regression with one explanatory variable.

References

1. K.R. Shah and B.K. Sinha, *Theory of Optimal Designs*, Lecture notes in Statistics, Springer-Verlag.
2. A.S. Hedayat, N.J.A. Sloane, John Stufken, *Orthogonal Arrays: Theory and Applications*, Springer.
3. A. Dey and R. Mukerjee, *Fractional Factorial Plans*, Wiley.
4. Andre I. Khuri and John A. Cornell*Response surfaces: designs and analyses*, CRC Press.
5. S. Gupta and R. Mukerjee, *A Calculus for Factorial Arrangements*, Lecture notes in Statistics, Springer-Verlag.
6. M Bose and A. Dey, *Crossover Designs*, World Scientific.
7. D. Raghavarao, *Constructions and Combinatorial Problems in Design of Experiments*, Wiley.

Advanced Functional Analysis

Prerequisite: Functional Analysis

- General Theory of topological vector spaces with emphasis to locally convex spaces. Linear Operators and functionals.
- Weak and weak* topologies on Banach spaces. Banach-Alaoglu Theorem. Goldstein's Theorem. The double dual. Reflexivity.
- Geometric Hahn-Banach theorem and applications. Extreme points and Krein-Milman theorem. Duality in Banach spaces.
- In addition, one of the following topics:
  - Geometry of Banach spaces: vector measures, Radon-Nikodym Property and geometric equivalents. Choquet theory. Weak compactness and Eberlein-Smulian Theorem. Schauder Basis.
  - Banach algebras, spectral radius, maximal ideal space, Gelfand transform.
  - Unbounded operators, Domains, Graphs, Adjoints, spectral theorem.

References

1. N. Dunford and J. T. Schwartz, *Linear Operators, Part II: Spectral Theory, Self Adjoint Operators in Hilbert Space*, Interscience Publishers, John Wiley (1963).
2. Walter Rudin, *Functional Analysis* Second edition, International Series in Pure and Applied Mathematics. McGraw-Hill (1991).
3. K. Yosida, *Functional Analysis*, Grundlehren der Mathematischen Wissenschaften (123), Springer- Verlag (1980).
4. J. Diestel and J.J. Uhl, Jr., *Vector Measures*, Mathematical Surveys (15), AMS (1977).

Advanced Multivariate Analysis

- Unified Approach for Constructions of Probability Distributions on Rp and on Smooth Manifolds - Torus and Sphere; Bivariate distributions based on Copula; Conditionally specified multivariate probability distributions: constructions, characterizations and inference; Multivariate Elliptically Contoured and Stable Families of Probability Distributions. Multivariate Mixture Probability Distributions.

- Decision-theoretic studies in simultaneous estimation problems with and without constraints; Pitman closeness and its applications to inadmissibility studies of multi-parameter estimators.
- Constructions of exact optimal tests for multiparameter simple hypotheses; Parameter Orthogonality, Invariance and Wijsman's representation of distribution of maximal invariant and their roles in the Construction of asymptotically optimal tests for multiparameter hypotheses in the presence of non/location-scale vector nuisance parameters.
- Dimension Reduction Techniques: Principal Component and Generalized Canonical Variable Analysis – Constructions and related Inference problems.
- Large p – Small n problems in Testing of Multiparameter Hypotheses: Likelihood and Union-Intersection Approaches; Tests for the mean vector in $N_p(\mu, \Sigma)$ and null and non-null asymptotic distributions of their test statistics, and MANOVA with $n < p$.
- Practical assignments based on the above topics will be part of the course.

References

1. Anderson, T.W. (1984). *An Introduction to Multivariate Statistical Analysis*. 2nd Edn. J. Wiley.
2. Kariya, T. and Sinha, B.K. (1989). *Robustness of Statistical Tests*. Academic Press.
3. Muirhead, R.J. (2009). *Aspects of Multivariate Statistical Theory*. J. Wiley
4. Seber, G.A.F. (2009). *Multivariate Observations*. Wiley.
5. Benjamini, Y., Hochberg, Y., 1995. Controlling the false discovery rate: a practical and powerful approach to multiple testing. *J. Roy. Statist. Soc. B* 57, 289–300.
6. Cambanis, S. , Huang, S. and Simons, G. (1981) On the theory of elliptically contoured distributions. *J. Multivariate Anal.* 11, 368-385.
7. Keating, J.P. and Mason, R.L. (1988). James-Stein estimation from an alternative perspective. *The American Statistician*. 42, 160-164.

Advanced Nonparametric Inference

- Density Estimation: Kernel-type density estimates and their asymptotic properties, optimal bandwidth selection.
- Nonparametric Function estimation: Adaptive estimation and minimax bounds for classes of functions, Oracle inequalities, model selection.
- Asymptotic justification of Jackknife and Bootstrap: Consistency of Jackknife Variance Estimates, Consistency of Bootstrap and Higher Order Accuracy of Bootstrap.
- Locally Most Powerful Rank Tests. Asymptotic theory of rank tests under null and alternative (contiguous) hypotheses, asymptotic relative efficiency.

References

1. Hajek, J., Sidak, Z. and Sen, P.K. *Theory of Rank Tests* (2nd Edition).
2. Serfling, R.J. *Approximation Theorems of Mathematical Statistics*.
3. Efron, B. and Tibshirani, R.J. *An Introduction to the Bootstrap*.
4. van der Vaart, A. *Asymptotic Statistics*.
5. Tsybakov, A.B, *Introduction to Nonparametric Estimation*.
6. Hall, P. *The Bootstrap and Edgeworth Expansion*.
7. Silverman, B.W. *Density Estimation for Statistics and Data Analysis*.
8. Efromovich, S., *Nonparametric Curve Estimation: Methods Theory, and Applications*.

Advanced Sample Surveys

Prerequisite: Sample Surveys and Design of Experiments

- Sufficiency, minimal sufficiency, Bayesian sufficiency in sampling, construction of complete class of estimators.
- Rao, Hartley and Cochran 's strategy. Admissibility among homogeneous linear unbiased estimators and also among all unbiased estimators.

- Godambe-Thompson's model-based design-unbiased estimators. Brewer-Royall's optimal predictors. Model-assisted asymptotically unbiased estimators. Generalized Regression (greg) estimators/ predictors and their accuracy estimators. Small Area Estimation. Post-stratification. Synthetic estimation.

- Replicated and repeated sampling, balanced repeated replication, Jack-knifing, bootstrap in finite population sampling.

- Randomized Response Techniques with qualitative and quantitative characteristics. Optional randomization and protection of privacy

- Network sampling. Adaptive sampling. Poisson's scheme of sampling and permanent random numbers.

- Organization of large-scale surveys. Familiarity with NSSO activities.

References

1. H. Bolfarine and S. Zacks (1992), *Prediction Theory for Finite Populations*, Springer-Verlag.

2. K.R.W. Brewer (2002), *Survey Sampling Inference*, Arnold.

3. Arijit Chaudhuri (2010), *Essentials of Survey Sampling*, PHI, Delhi.

4. Arijit Chaudhuri (2011), *Randomized Response and Indirect Questioning Techniques in Surveys*, Chapman and Hall.

5. Arijit Chaudhuri (2014), *Modern Survey Sampling*, Chapman and Hall.

6. Arijit Chaudhuri (2014), *Network and Adaptive Sampling*, Chapman and Hall.

Analysis of Directional Data

- Examples, interpretations and summary statistics for directional data.

- Constructions of Probability Distributions on Circle, Hyperdisc and Smooth Manifolds. General Fourier series representations of single and doubly periodic functions for probability distributions on circle and torus. Mobius transformations in the complex domain and directional probability distributions. Levy-Lucas theorem and Wrapped Symmetric Stable Mixture Family of Circular distributions. Maxent characterizations of circular probability distributions.

- ML and TMM estimation of parameters of directional distributions; Robust Optimality Invariant tests for Isotropy; Wintner's theorem and its applications to monotonicity of power functions of circular tests; Analysis of Mean Directions; Tests for Homogeneity of circular concentration parameters; Tests for Independence of random linear/circular and circular components of a directional vector random variable; Circular Goodness-of-Fit tests: K-S and Watson's generalization of Cramer -von Mises functional tests.

- Regression Analysis for directional data: Toroidal and Cylindrical regressions.

- Introduction to Classification and Cluster analysis with directional data.

- Introduction to Bayesian analysis and inference for directional data – applications of Conjugate Priors, Dirichlet Process and Hierarchical Bayes approach.

- Practical assignments based on the above topics will be part of the course.

References

1. Mardia, K.V. (1972). *Statistics of Directional Data*. Academic Press. London.

2. Jammalamadaka, S.R. and SenGupta, A. (2001). *Topics in Circular Statistics*. World Scientific. New Jersey.

3. Watson, G.S. (1983). *Statistics on Spheres*. J. Wiley. New York.

4. Bagchi, P. and Kadane, J.B. (1991). Laplace approximations to posterior moments and marginal distributions on the circles, spheres and cylinders. *The Canadian J. Statist.* 19, 67-77.

5. Larsen, P. V. , Blaesild, P. and Sorensen, M. K. (2002). Improved likelihood ratio tests on the von Mises – Fisher distribution. *Biometrika* 89, 947-951.

6. Singh, H. , Hnizdo. V. and Demchuk, E. (2002). Probabilistic model for two dependent circular variables. *Biometrika* 89, 719-723.

Asymptotic Theory of Inference

- General overview of consistency of estimators with emphasis on consistency of maximum likelihood estimates.

- Contiguity. Local asymptotic normality, differentiability in quadratic mean, asymptotic efficiency of estimators.

- Brief overview of Vapnik-Chervonenkis Theory for Empirical Processes (with only motivation and statement of theorems without proofs).
- Functional delta method with applications.
- M and Z estimators, argmax theorem, applications.
- Semiparametric models and methods, standard estimation approaches through likelihood and estimating equations. Tangent spaces and information, score and information, semiparametric efficiency.

References

1. van der Vaart, A.W. *Asymptotic Statistics*.
2. Ibragimov, I.A. and Has'minskii, R.Z. *Statistical Estimation: Asymptotic Theory*.
3. Roussas, C.G. *Contiguity of Probability Measures*.
4. LeCam, L.M. and Yang, G. *Asymptotics in Statistics*.
5. Bickel, P.J., Klaassen, C.A.J., Ritov, Y. and Wellner, J.A. *Efficient and Adaptive Estimation for Semiparametric Models*.
6. Kosorok, M.R. *Introduction to Empirical Processes and Semiparametric Inference*.
7. Begun, J.M. , Hall, W.J. , Huang, W.M. and Wellner, J.A. (1983), *Annals of Statistics*, 432-452. Information and Asymptotic Efficiency in Parametric-Nonparametric Models.
8. Ritov, Y. and Bickel, P.J. (1990), *Annals of Statistics*, 925-938. Achieving information bounds in non and semiparametric models.

Bayesian Computation

Prerequisite: *Statistical Inference II*

- Deterministic methods: Numerical integration methods for computation in moderately high- dimensional posteriors, normal approximation, Edgeworth expansion, saddle-point approximation, Laplace approximation, nested Laplace approximation, theoretical results on the errors of approximation, dynamic programming method of exact mode-finding for chain structured Bayesian models.

- Direct simulation methods: Random variable generation using accept-reject methods, the squeeze principle, adaptive rejection sampling for log-concave densities, Importance Sampling and Resampling methods, theory on efficiency of Importance Sampling methods, comparison of importance sampling with accept-reject methods, controlling Monte Carlo variance – Rao- Blackwellization, antithetic variables, control variates, monitoring convergence of Monte Carlo methods.

- MCMC methods: Theory of finite Markov chains and their convergence, general state space Markov chains, detailed convergence theory, Gibbs sampler, Metropolis-Hastings sampler, detailed convergence properties of these samplers, small sets, drift conditions, geometric and uniform ergodicity, mixtures and cycles of Markov kernels, optimal scaling of random walk and Langevin based Metropolis-Hastings algorithms, slice sampler, convergence properties of the slice sampler, Hybrid Monte Carlo, variable dimensional problems and reversible jump MCMC methods, rigorous simulated annealing theory and methods for stochastic optimization.

- MCMC convergence diagnostics: Exploratory methods, nonparametric tests, rigorous methods based on regenerative simulation, introduction to the theory of perfect simulation.

- Iterated and Sequential Importance Sampling: Importance sampling, generalized importance sampling; Particle Systems – Sequential Monte Carlo, Hidden Markov Models, Weight Degeneracy, Particle Filters, Sampling Strategies, Fighting the Degeneracy, Convergence of the Particle Systems, Population Monte Carlo, Dynamic Importance Sampling.

- Bayes factor computation: Bayes Information Criterion, Laplace approximation, Umbrella sampling, Thermodynamic integration, Acceptance ratio method, Bridge sampling, path sampling methods, optimality of the paths, variable-dimensional methods.

- Doubly intractable problems: Approximate Bayes Computation (ABC) method, combination of MCMC and importance sampling, bridge-exchange algorithms.

- Software: BUGS, JAGS, LaplacesDemon.

References

1. Robert and Casella, *Monte Carlo Statistical Methods* (Springer).

2. K. Lange, *Numerical Analysis for Statisticians* by (Springer).

3. Richardson and Spiegelhalter, *Markov Chain Monte Carlo in Practice by Gilks*, (Chapman and Hall).

4. J. Liu, *Monte Carlo Strategies in Scientific Computing* (Springer).

5. S. Meyn and R. Tweedie, *Markov Chains and Stochastic Stability* (Cambridge University Press).

6. P. Bremaud, *Markov Chains – Gibbs Fields, Monte Carlo Simulation, and Queues* (Springer).

7. Brooks, Gelman, Jones and Meng *Handbook of Markov Chain Monte Carlo* (Cambridge University Press).

Branching Processes

Prerequisite Martingale Theory

- Galton-Watson Branching Processes: Review of the classical branching process, definition, generating functions, extinction probability, sub-critical, critical and super-critical phases.

- Kesten-Stigum Theorem and strong convergence in super-critical case. Conditional limit theorems: sub-critical case: Yaglom's Theorem; critical-case: Kolmogorov's Theorem. Decomposition of the super-critical branching process. Second order properties of Zn /mn.

- One Dimensional Continuous Time Markov Branching Processes: Definition, construction, generating functions. Extinction probability and moments. Examples: Binary Fission, Birth and Death processes. The embedded Galton-Watson process and applications. Limit theorems, split times. Second order properties. The embeddability problem.

- Multi-Type Branching Processes: Definitions and examples. Moments and Frobenius Theorem. Extinction probability and transience. Limit theorems for sub, super and critical cases. Introduction to continuous time multi-type Markov branching processes.

- (If time permits) Age dependent processes and Embedding of Urn Schemes into Continuous Time Markov Branching Processes (Athreya-Karlin Embedding).

References

1. K.B. Athreya and P.E. Ney, *Branching Processes*.

2. S. Asmussen and H. Hering, *Branching Processes*.

3. T.E. Harris, *The Theory of Branching Processes*.

4. C.J. Mode, *Multitype Branching Processes*.

Commutative Algebra

Prerequisite: Abstract Algebra

- Rings and ideals: review of ideals in quotient rings; prime and maximal ideals, prime ideals under quotient, existence of maximal ideals; operations on ideals (sum, product, quotient and radical); Chinese Remainder theorem; nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms; prime avoidance.

- Modules over commutative rings: submodules and quotient modules, homomorphisms, direct summand, product, free modules, exact sequences. Tensor products of modules and algebras. [8] Localisation (Rings and modules of fractions) and local rings, extended and contracted ideals under localisations localisation, localisation and quotients, exactness property.

- Modules over local rings. Cayley-Hamilton, NAK lemma and applications. Examples of local-global principles.

- Chain Conditions, Noetherian Rings and Modules. Hilbert's Basis Theorem. Associated Primes and Primary Decomposition. Artininan Modules. Modules of Finite Length.

- Integral Extensions: integral closure, normalisation and normal rings. Cohen-Seidenberg GoingUp Theorem. Hilbert's Nullstellensatz and applications.

- Discrete valuation rings and Dedekind domains.

References

1. N.S. Gopalakrishnan: *Commutative Algebra*.

2. M.F. Atiyah and I.G. Macdonald: *Introduction to Commutative Algebra*.

3. M. Reid: *Undergraduate Commutative Algebra*, LMS Student Texts (29).

4. R.Y. Sharp: *Steps in Commutative Algebra*, LMS Student Texts (19).

5. Balwant Singh: *Basic Commutative Algebra*.

6. D.S. Dummit and R.M. Foote: *Abstract Algebra* (Part V).

Descriptive Set Theory

- A quick review of elementary cardinal and ordinal numbers, transfinite induction, induction on trees, Idempotence of Souslin operation.
- Polish spaces, Baire category theorem, Transfer theorems, Standard Borel spaces, Borel isomorphism theorem, Sets with Baire property, Kuratowski-Ulam Theorem. The projective hierarchy and its closure properties.
- Analytic and coanalytic sets and their regularity properties, separation and reduction theorems.
- von Neumann and Kuratowski-Ryll Nardzewskis selection theorems, Uniformization of Borel sets with large and small sections. Kondos uniformization theorem.

References

1. S.M. Srivastava, *A Course on Borel Sets*, GTM (180), Springer-Verlag (1998).

2. A.S. Kechris, *Classical Descriptive Set Theory*, GTM (156), Springer-Verlag (1995).

Ergodic Theory

Prerequisite: Measure Theoretic Probability

- Statistical Mechanics background for measure-preserving transformation, Liouville's theorem.
- Different examples of measure-preserving transformation; Ergodicity.
- Ergodic theorems
- Kakutani's Tower, Rokhlin's Lemma; Induced transformation.
- Mixing (weak and strong): characterizations, examples.
- Weak mixing of multiple order, Furstenberg multiple recurrence theorem for weak mixing transformation, Application to Szemeredi's theorem

- Isomorphism, Conjugacy and Spectral isomorphism
- Measure-preserving transformation with discrete spectrum, Eigenvalues and eigenfunctions, Group orations, Halmos-von Neumann representation
- Entropy: Entropy of a partition, entropy of a measure-preserving transformation, methods of calculating entropy, Kolmogorov-Sinai theorem, *Shannon-Mc-Millan-Breiman theorem, Bernoulli automorphism, Kolmogorov automorphism.

References

1. Peter Walters, *An Introduction to Ergodic Theory*, GTM (79), Springer-Verlag (1982).
2. Patrick Billingsley, *Ergodic Theory and Information*, Robert E. Krieger Publishing Co. (1978).
3. M.G. Nadkarni, *Basic Ergodic Theory*, TRIM 6, Hindustan Book Agency (1995).
4. H. Furstenberg, *Recurrence in Ergodic Theory and Combinatorial Number Theory*, Princeton University Press (1981).
5. K. Petersen, *Ergodic Theory*, Cambridge Studies in Advanced Mathematics (2), Cambridge University Press (1989).

Fourier Analysis

Prerequisite: Analysis II or Metric Topology and Complex Analysis

- Fourier and Fourier-Stieltjes' series, summability kernels, convergence tests. Fourier transforms, the Schwartz space, Fourier Inversion and Plancherel theorem. Poisson summation formula, Paley- Wiener Theorem. Heisenberg uncertainty Principle, Wiener's Tauberian theorem.
- Maximal functions and boundedness of Hilbert transform.
- Introduction to wavelets and multi-resolution analysis.

References

1. Y. Katznelson, *An Introduction to Harmonic Analysis*, Dover Publications (1976).
2. E.M. Stein and Rami Shakrachi, *Fourier Analysis: An Introduction*, Princeton Lectures in Analysis.
3. E. Hernez and G. Weiss, *A First Course on Wavelets*, Studies in Advanced Mathematics. CRC Press (1996).

General Topology

Prerequisite: Analysis II or Metric Topology and Complex Analysis

- Topological Spaces: Definition and examples, Open and closed sets, metrizable spaces, relative topology, subbases and bases, dense sets, closure and interior, boundary, isolated and limit points of a space, continuous functions and its various equivalent definitions.

  Countability Axioms: first countable and second countable spaces, Separable spaces.

- Separation Axioms: T_1, T_2, T_3, $T_{3\frac{1}{2}}$ and T_4 spaces, normality of metrizable spaces, Urysohn lemma and Tietze extension theorem for normal spaces.

- Covering Axioms: Lindelof spaces, Urysohn theorem on normality of regular Lindelof spaces, Compactness and its various equivalent definitions, Continuous function on compact spaces, Locally Compact spaces.

- Product Spaces: Product topology and continuity, countable product of metric spaces, Urysohn embedding lemma, metrization theorem for second countable spaces, Tychonoff's theorem.

- Connected spaces: connected and locally connected spaces, path connected and locally path connected spaces, connected component, connectedness and product topology.

- Spaces of the First Category and Second Category spaces, Baire category theorem for complete metric spaces and for locally compact Hausdorff spaces, some applications.

- Quotient topology: Definition, computation of some standard quotient spaces such as S^1, Mobius strip, Torus, Klein's bottle and projective spaces.

- Homotopy of Paths, Fundamental Groups, Covering Spaces, The fundamental groups of the Circle, Punctured Plane, S^n, Projective Plane P^2, Torus T, Double Torus T^2.

References

1. J.R. Munkres: *Topology*.

2. M.A. Armstrong: *General Topology*.

3. G.F. Simmons: *An Introduction to Topology and Modern Analysis*.

4. S.M. Srivastava: *A Course on Borel Sets*.

Life Testing and Reliability

- Coherent Systems: System of components, Coherent structure, Representation of coherent structures in terms of paths and cuts, Relative importance of components, Modules of coherent structures, Event trees for complex systems.

- Reliability of Coherent Systems: Reliability of systems of independent components, Reliability importance, Association of random variables, Bounds on system reliability, Shape of the system reliability functions.

- Parametric Families of Distributions in Reliability: A notion of aging, The exponential distribution, Applications of Poisson processes, Spare parts problem, Life distributions with monotone failure rate, Estimation from parametric distributions with censored data.

- Classes of Life Distributions: Life distribution of coherent systems, IFRA distribution arising from shock models, Preservation of life distribution classes under reliability operations, Partial orderings of life distributions, Reliability bounds, Mean life of series and parallel systems.

- Concepts in Maintenance/Replacement Policy: Distributions in replacement policy, Renewal theory in replacement models, Replacement policy comparisons, preservation of life distribution classes.

- Maintenance and Replacement Models: Availability theory, Maintenance through spares and repair.

- Two Dual Types of Failures.

- Multivariate Life Time Distributions: Bivariate exponential distribution, Multivariate exponential distribution, Multivariate monotone failure rate distributions.

References

1. R.E. Barlow and F. Proschan, *Statistical Theory of Reliability and Life Testing*, 1975.

2. H. Ascher and H. Feingold, *Repairable Systems Reliability*, 1984.

Markov Processes and Martingale Problems

Prerequisite: Brownian Motion and Diffusions

- Semigroup Theory: Operator semigroups, strongly continuous contraction semigroups, Generators. Basic properties. Resolvent operators, Hille – Yosida Theorem.

- Markov Processes: Definitions. Markov property, Transition functions and Chapman Kolmogorov property, Ionescu-Tulcea theorem. Associated Semigroups. Examples. Strong Markov Property. Jump Processes. Feller property. Hille – Yosida Theorem for positive semigroups. Equivalence of martingales associated with Brownian motion. Martingales associated with Markov processes.

- Martingale Problems: Definitions. Path properties of solutions. Uniqueness. Martingale problems as characterization of Markov Processes. Piecing of solutions. Convergence results. Perturbations of Generators.

- Equivalence between martingale problems and SDE's. Ito processes. Super processes.

References

1. S.N. Ethier and T.G. Kurtz, *Markov Processes: Characterization and Convergence* (1986).

2. D.W. Stroock and S.R.S. Varadhan, *Multidimensional Diffusion Processes* (1971).

3. J. Jacod, *Calcul Stochastique et Promlemes de Martingales* (1979).

Mathematical Biology

- Linearization of dynamical systems (two, three, and higher dimensions), Stability theory: (a) asymptotic stability (Hartman's theorem), (b) Global stability (Liapunov's direct method), Translation property, limit sets, attractors, periodic orbits, limit cycles and separatrix, Bendixon criterion, Dulac criterion, Poincare-Bendixon theorem, Bifurcation: saddle-node, transcritical, pitchfork, Hopf.

- Single, and multispecies population growth models, predator-prey models, competition models, models on mutualism, food chain models, time delay models, phytoplankton-zooplankton models.

- Fick's law, Turing pattern, diffusion driven instability, population dynamics models with self and cross diffusion.

- Deterministic, and stochastic models on simple epidemics, general epidemics, pure birth-death process, simple models on spatial spread of epidemics, recurrent epidemics models. models on malaria, HIV, AIDS, Dengue.

- Basic concepts on eco-epidemiological systems.

References

1. D.N. Jordan and P. Smith (1998): *Nonlinear Ordinary Equations–An Introduction to Dynamical Systems* (3rd ed).
2. L. Perko (1991): *Differential Equations and Dynamical Systems*.
3. H.I. Freedman (1990): *Deterministic Mathematical Models in Population Ecology*, Pure and Applied Mathematics Series, Vol. 57, Mercel Dekker.
4. Mark Kot (2001): *Elements of Mathematical Ecology*.
5. J.D. Murray (1990): *Mathematical Biology*.
6. Eric Renshaw (1990): *Modelling Biological Population in Space and Time*.
7. Leah Edelestin-Keshet (2005): *Mathematical Models in Biology*.
8. N.T.J. Bailey (1975): *The Mathematical Theory of Infectious Diseases and its Applications*.
9. Roy M Anderson and Robert M May (1991): *Infectious Diseases of Humans: Dynamics and Control*.
10. Horst Malchow, Sergei V Petrovoski, and Ezio Venturino (2008): *Spatiotemporal Patterns in Ecology and Epidemiology: Theory, Models and Simulations*.

Ordinary and Partial Differential Equations

- Linear ODE.
- Power series method and orthogonal polynomials.
- Picard's theorem, generalities of PDE.
- Heat, Laplace and wave equations.
- Initial value problems.
- Boundary value problems.

References

1. G.F. Simmons, Differential Equations, McGraw-Hill.

2. G.B. Folland, Introduction to partial differential equations, Princeton University Press (1995).

3. F. Trs, *Basic Linear Partial Differential Equations, Pure and Applied Mathematics* (62), Academic Press (1975).

4. J. Rauch, *Partial Differential Equations*, GTM (128), Springer-Verlag (1991).

5. E. DiBenedetto, *Partial Differential Equations*, Birkhauser (1995).

6. L.C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics (19), AMS (1998).

7. L. Hormander, *The Analysis of Linear Partial Differential Operators. I. Distribution Theory and Fourier Analysis*. Grundlehren der Mathematischen Wissenschaften (256), Springer-Verlag (1990).

Percolation Theory

- Introduction to bond and site percolation. Formal probability set-up. Critical phenomenon and its existence.
- FKG inequality, BK inequality (only for increasing events), Russo's formula.
- Exponential decay of the percolation probability below criticality.
- Uniqueness of the infinite open cluster.
- Critical probability for two dimensions is $\frac{1}{2}$.
- Oriented percolation in two dimensions: Subadditive ergodic theory; introduction and the model; characterisation of pc. Recurrence properties of the right edge process. Exponential estimates for $p < pc$. Proof no percolation at criticality. Exponential decay of time of 'extinction'.
- Continuum percolation: The Boolean model; Coupling and scaling, FKG inequality; Occupancy in Boolean models; Vacancy in Boolean models, the covered volume fraction.

References

1. G. Grimmett, *Percolation* 2nd edition, Springer (1999).

2. T. Liggett, *Interacting Particle Systems*, Springer.

3. R. Durrett, Oriented percolation in two dimensions, *Ann. Probab.* 1984, 999-1040.

4. R. Meester and R. Roy, *Continuum Percolation*, Cambridge (1996).

Random Walks and Electrical Networks

- Finite Networks: Review of the random walks on finite subsets of one and two dimensional lattices. Discrete harmonic functions on one dimension and uniqueness. Harmonic functions in two dimensions, Dirichlet problem.

- Random walks on general finite networks. General electrical networks and reversible Markov chains. Definition of voltage and current and their probabilistic interpretations. Kirchhoff's Node Law, Cycle Law and Ohm's Law. Effective resistance and effective conductance. Relation with escape probability. Thomson's Principle. Rayleigh's Monotonicity Law.

- Infinite Networks: Review of random walks on infinite integer lattice, recurrence, transience and Polya's Theorem (recurrent on line and plane but transience on space). Escape probability for infinite network and electrical network formulation. Definition of effective resistance.

- Network reduction: Rayleigh's shorting and cutting laws. Recurrence for SSRW in one and two dimensions. Transience for random walk on d-array trees. Transience for SSRW in dimension three and above.

- Random walks on general graph and type problem for a graph. Comparison of two infinite electrical networks. Flows, cut-sets and random paths. The Max-Flow Min-Cut Theorem. Comparing general graphs with lattice graphs, solution to the type problem.

References

1. P.G. Doyle and J.L. Snell, *Random Walks and Electrical Networks*.

2. Y. Peres and R. Lyons, *Probability on Trees and Networks* (Chapters 2 and 3).

3. S. Coste, *Lecture Notes on Finite Markov Chains*.

4. G. Lawler and V. Limic, *Random Walk: A Modern Introduction*.

5. A. Telcz, *Art of Random Walks*.

6. W. Woess, *Random Walk on Infinite Groups and Graphs*.

Representation Theory of Finite Groups

- Representations of finite groups: Concept of representation.
- Complete reducibility, uniqueness of decomposition. Group ring and regular representation, space of class functions, orthogonal relations.
- Induced characters, induced representations, positive decomposition of regular character, Brauer's theorem.

References

1. C.W. Curtis and I. Reiner, *Representation Theory of finite Groups and Associative Algebras*, Springer.
2. P.M. Cohn, *Further Algebra and Applications*, Springer.
3. J.P. Serre. *Linear Representations of Finite Groups*.

Resampling Techniques

Same as Resampling Techniques of M.Stat. First Year; available only to those students who have not taken the course in M. Stat. First Year.

Risk Management

Prerequisite: Quantitative Finance

- Loss operators, risk measures, Estimating VaR and Expected Shortfall, Exact and bootstrap confidence intervals.
- Extreme value theory methods for risk management: distribution of maxima, modelling tails, measures of tail risk, Hill estimation, Point process models, Peaks-over-threshold.
- Elliptical distributions and Copulas: Basics; measures of dependence. Gaussian, elliptical, t and Archimedean copulas; fitting copulas to data, extreme value copulas, threshold copulas and their limits.
- Market risk: Foreign exchange futures, forwards, swaps and options. Pricing relationships and contract structures. Use of the yield curve for pricing cash flows. Measuring risk exposures and using derivatives to manage risks.
- Liquidity risk: Market Liquidity and funding liquidity risk, Liquidity spiral.

- Credit risk: events of default, default probabilities. Structural models: Merton and KMV models. Bond models: credit spreads. Correlations and default dependencies. Estimating losses given default. Copula approaches to estimating defaults. Portfolio models of credit risk. Stress testing. Managing credit risks using credit derivatives. Limitations and risks of credit derivatives.

References

1. Henrik Hult and Filip Lindskog: *Mathematical Modelling and Statistical Methods for Risk Management*.
2. Alexander J. McNeil, Rüdiger Frey and Paul Embrechts: *Quantitative Risk Management: Concepts, Techniques, and Tools*.
3. Robert E. Whaley: *Derivatives: Markets, Valuation, and Risk Management*.
4. Darrell Duffie and Kenneth J. Singleton: *Credit Risk: Pricing, Measurement and Management*.

Robust Statistics

- The need for robust methods; statistical functionals; measures of robustness, sensitivity curves and influence functions; robustness in location and scale models.
- M-estimators, asymptotic distribution of M-estimators, re-descending M-estimators, tuning an estimate, L-estimators.
- Max-bias and breakdown point; robustness based on data depth.
- Robust regression; M-estimators, GM-estimators, Mallows type estimator, Least median of squares.
- Multivariate Data; Multivariate M-estimators; Minimum Volume Ellipsoid. Computational issues; iteratively reweighted least squares and other methods.
- Distance based methods; minimum power divergence and minimum density power divergence estimators, minimum Hellinger distance estimator.

References

1. Huber, P.J. And Ronchetti, E.M. (1981). *Robust Statistics* (2nd Ed). Wiley, New York.

2. Hampel, F.R. , Ronchetti, E.M. , Rousseeuw, P.J. and Stahel, W.A. (1986). *Robust Statistics: The Approach Based on Influence Functions.* Wiley, New York.

3. Maronna, R.A. , Martin, R.D. and Yohai, V.J. (2006). *Robust Statistics: Theory and Methods.* Wiley, New York.

4. Rousseeuw, P.J. and Leroy, A.M. (1997). *Robust Regression and Outlier Detection*. Wiley, New York.

5. Basu, A. Shioya, H. and Park, C. (2011). *Statistical Inference: The Minimum Distance Approach*. CRC Press, Boca Raton, Florida.

6. Serfling, R. (1980). *Approximation Theorems of Mathematical Statistics*. Wiley, New York.

Signal and Image Processing

Signal Processing

- Discrete-time signals and systems, Linear Time Invariant (LTI) Systems and their properties.
- Frequency domain representation: The Discrete-Time Fourier Transform (DTFT) and its properties.
- Discrete-time random signals and their application in LTI systems.
- The z-transform and its properties.
- Sampling of continuous-time signals, Nyquist's Sampling Theorem.
- Frequency response of systems with rational system functions, all-pass systems, minimum-phase systems, systems with linear phase.
- The Discrete Fourier Transform and its properties, sampling the Fourier Transform.
- Structures for discrete-time systems, block diagram representations.
- DFT computation –the decimation-in-time FFT.
- Introduction to Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) Filter Design.

Image Processing

- Digital image fundamentals: digital image representation and creation by sampling and quantization; resolution and detail; properties of pixels.
- Typical IP operations like enhancement, contrast stretching, smoothing and sharpening, grey-level thresholding, edge detection, medial axis transform, skeletonization/ thinning.
- Statistical models for digital images and image restoration methods based on them (e. g. , Besag (1986), Geman and Geman (1984), Amit, Grenander and Piccioni (1991)).
- Segmentation: detection of discontinuities, boundary detection, thresholding, clustering, region-based object representation/description and recognition.
- Overview of image compression: standard lossy and lossless methods.
- Introduction to Colour image processing, Wavelets and multiresolution processing and Morphological image processing.

References

1. A.V. Oppenheim, R.W. Schafer and J.R. Buck, *Discrete-time Signal Processing*, Prentice-Hall.
2. J.G. Proakis and D.G. Manolakis, *Digital Signal Processing*, Pearson-Prentice Hall.
3. S.K. Mitra, *Digital Signal Processing – A Computer-Based Approach*, Tata McGraw Hill.
4. R.C. Gonzalez and R.E. Woods, *Digital Image Processing* (3rd edition), Pearson Education India, 2011.
5. P. Fieguth, *Statistical Image Processing and Multidimensional Modeling*, Springer, 2011.
6. S. Jayaraman, S. Esakkirajan and T. Veerakumar, *Digital Image Processing*, Tata McGraw-Hill, 2009.
7. W.K. Pratt, *Digital Image Processing*, Wiley, 2007.
8. K.R. Castleman, *Digital Image Processing*, Pearson Education India, 2007.
9. K.V. Mardia and G.K. Kanji, *Statistics and Images*. Carfax, 1993.
10. A. Jain, *Fundamentals of Digital Image Processing*. Prentice-Hall, 1989.

Special Topics in Economics

From time to time, an optional course on a topic of current interest in Economiics, which is not covered by any of the optional courses listed above, may be offered, if a faculty-member (permanent/ visiting) wishes to offer such a course. However, a brief description of the topics to be covered in the course has to be submitted by the concerned teacher and approval of the Academic Council has to be obtained in advance.

Special Topics in Finance

From time to time, an optional course on a topic of current interest in Finance, which is not covered by any of the optional courses listed above, may be offered, if a faculty-member (permanent/ visiting) wishes to offer such a course. However, a brief description of the topics to be covered in the course has to be submitted by the concerned teacher and approval of the Academic Council has to be obtained in advance.

Special Topics in Probability

From time to time, an optional course on a topic of current interest in Probability, which is not covered by any of the optional courses listed above, may be offered, if a faculty-member (permanent/visiting) wishes to offer such a course. However, a brief description of the topics to be covered in the course has to be submitted by the concerned teacher and approval of the Academic Council has to be obtained in advance.

Special Topics in Statistics

From time to time, an optional course on a topic of current interest in Statistics, which is not covered by any of the optional courses listed above, may be offered, if a faculty-member (permanent/ visiting) wishes to offer such a course. However, a brief description of the topics to be covered in the course has to be submitted by the concerned teacher and approval of the Academic Council has to be obtained in advance.

Statistical Methods in Demography

- Sources of Demographic Data: Populations: open and closed, de facto and de jure, censuses and population registers, lexix diagram and classification of events, register data and epidemiologic studies, sampling in censuses and dual system estimation.

- Mortality Analysis and Models: Measures of mortality, comparing mortality experiences, life table functions, multistate life tables, duration-dependent life tables, hazards and survival probabilities, life expectancies and stable populations, Kaplan-Meier and Nelson-Aalen estimators, estimation of survival proportions, regression models (generalised linear models, binary regression models, Poisson regression models) for counts and survival, Brass logit survival model, Lee-Carter mortality model.

- Fertility Analysis and Models: Indices and rates of fertility, comparing fertility experiences, period and cohort analysis of fertility, indirect estimation of fertility, analysis of nuptiality, parity progression, determinants of fertility, Bongaarts and Potter's model, components of birth interval and its analysis.

- Population Growth: Linear growth model, matrix formulation, stable populations, weak ergodicity, open populations and parametrization of migration, demographic functional, Markov chain models.

- Models of Population Structure: Age and sex structure of population, demographic determinants of the shape of population pyramid, stationary and stable populations, stable population's fertility, mortality and age structure, length of a generation, model life tables, demographic reconstruction with two censuses.

- Population Projection: Trends, random walks and volatility in forecasting demographic rates, linear stationary processes, ARIMA models, integrated processes, differencing, regression and structural models for handling of non-constant mean, heteroscedastic innovations.

- Health Statistics: Measures of morbidity, prevalence and incidence rates, measures of risk, measures of association in cross sectional sampling, cohort and case-control studies, estimation of odds ratio, statistical inference on odds ratio including interval estimation, analysis of several 2X2 contingency tables, test of homogeneity, significance test of common odds ratio including finding out its confidence interval, estimating prevalence, estimating positive and negative predictive value, multiple decrement life tables.

- There should be practical exercises.

References

1. Alho, Juha M. and Spencer, B.D., *Statistical Demography and Forecasting*. 2005. Springer Series in Statistics.

2. Preston, Samuel, Heuveline, Patrick and Guillot, Michel, *Demography: Measuring and Modelling Population Processes*, Wiley-Blackwell.

3. Hinde, Andrew, *Demographic Methods*. 1998. Arnold.

4. Shoukri, M. M. and Pause, C. A., *Statistical Methods for Health Sciences*. 1999. CRC Press.

5. Hartman, Michael, *Demographic Methods for the Statistical Office*. 2009. Statistics Sweden, Research and Development.

6. Lawless, J.F., *Statistical Models and Methods for Lifetime Data*. 2003. Wiley-Interscience.

Statistical Methods in Epidemiology and Ecology

- Introduction to dynamical models in ecology and epidemiology, Introduction to parametric growth models, Single species growth models -exponential, logistic, extended logistic, Gompertz etc. , notion of density dependent and independent growth, asymmetry in growth dynamics, the notion of growth rate metric and its extension, distribution of growth rate and its asymptotics.

- Effect of measurement errors on growth rate and related inference problems, longitudinal data and growth curve analysis, goodness-of-fit test for growth curve models, profile likelihood, nonlinear growth models and asymptotics, resampling techniques in growth curves.

- Stochastic extension of growth models, concept of demographic and environmental stochasticity, notion of stochastic stability and related statistical diagnostics in population dynamics.

- Concepts of equilibrium and quasi equilibrium distribution and its moments, concept of Allee effects and association extinction dynamics, simple extension to interactive population dynamics.

- Mathematical models of infectious disease in stochastic environment, concept of stochastic SI, SIR, SIS epidemic models, estimation of basic reproduction number and time to extinction of disease, likelihood based inferences.

References

1. Michael J. Panik, *Growth Curve Modelling: Theory and Applications*, Wiley, 2015.

2. M. Henry H. Stevens, *A Primer in Ecology with R*, Springer, 2009.

3. Benjamin M. Bolker, *Ecological Models and Data in R*, Princeton university press, 2006.

4. F. Courchamp, L. Berec, and J. Gascoigne, *Allee Effects in Ecology and Conservation*, 2008.

5. Ludwig and Reynolds, *Statistical Ecology: A Primer in Methods and Computing*, John Wiley and Sons, 1988.

6. Seber and Wild, *Nonlinear Regression*, Wiley series in probability and statistics, 2003.

Stochastic Calculus for Finance

Available only to those students who have not taken Martingale Theory; students opting for this course need to also take Quantitative Finance.

- Probability basics: information and sigma-algebras, independence, conditional expectations, filtration, martingales, random walk, stopping times, Markov processes.

- Brownian Motion (BM): scaled random walks, definition of BM, finite dimensional distributions, filtration for BM, martingale property, quadratic variation, Markov property, local martingale and Levy's characterization, reflection principle, first passage time distribution, maximum process, geometric BM.

- Stochastic integration: Itô's integral for simple integrands and its extension to wider classes of integrands, isometry and martingale properties of Itô's integral, Itô-Doeblin formula, multivariable stochastic calculus

- Evolution of option price, deriving the Black-Scholes-Merton PDE, hedging, Greeks, put-call parity.

- Risk-neutral pricing: Girsanov's theorem, risk-neutral measure, deriving Black-Scholes-Merton formula, martingale representation, dividend paying assets, forwards and futures.

References

1. S. Shreve, *Stochastic Calculus for Finance II (Continuous-Time Models)*.

2. T. Bjork, *Arbitrage Theory in Continuous Time*.

3. Mikosch, *Elementary Stochastic Calculus with Finance in View*.

Theory of Extremes and Point Processes

Prerequisite: Measure Theoretic Probability

- Local uniform convergence of real-valued functions on real line, inverses of monotone functions, convergence to types theorem, univariate extreme value distributions and Fisher-Tippett-Gnedenko Theorem.

- Regularly varying functions of a real variable and their properties: Karamata's theorem and Potter's bounds. Domain of attraction of Frechet distribution.

- Quick review of weak convergence of probability measures on Polish spaces (without proofs): Portmanteau theorem, Skorohod's theorem, continuous mapping theorem, Prokhorov's theorem, Slutsky's theorem and converging together theorems.

- Fundamentals of point processes and random measures, Laplace functionals, Poisson processes: definition, construction, transformations, marking and thinning.

- Vague convergence on locally compact second countable Hausdorff topological spaces. Weak convergence of point processes and random measures.

- Application of point processes and random measures to extreme value theory: Hill estimator and its consistency. Second order regular variation and asymptotic normality of Hill estimator.

References

1. Resnick, *Extreme Values, Regular Variation and Point Processes*.

2. Resnick, *Heavy-Tail Phenomena: Probabilistic and Statistical Modelling*.

3. Kallenberg, *Random Measures*.

Theory of Games and Statistical Decisions

- Introduction: Games and Solutions, Statistical decision making as a game, Theory of Competitive Equilibrium, Rational Behaviour, Role of information in decision making (mostly with examples), Perfect, Complete information (examples).

- Strategic games: Examples, Nash Equilibrium, Existence of Nash Equilibrium, Two person noncooperative games, Saddle point, Minimax theorem for matrix game, Equivalence of Matrix games and Linear programming. General n-person finite games, Nash's theorem, Special emphasis to solving bimatrix games as Quadratic programming.

- Extensive games with perfect/imperfect information, application to examples from sequential analysis (when perfect information is not available, examples). Interpretation of a strategy, Examples from clinical trials, Bandit problems as game (finite horizon).
- Coalitional games: The Core, Interpretation of Bayesian Inference with many players (common prior theory), Stable sets, Bargaining sets, Shapely value.
- Bargaining problems and problems of "trade off" in statistical decision making, Nash solution, Discussion and Examples.
- Additional mathematical theories if any, like Markov decision problems, Stochastic games etc.

References

1. Vorob'ev, N.N., *Game Theory: Lectures for Economists and Systems Scientists*.
2. Karlin, S. , *Mathematical Methods and Theory in Games, Programming, and Economics* (volume 2).
3. Blackwell, D. and Girshick, M.A., *Theory of Games and Statistical Decisions*.
4. Kuhn, H.W. (Ed.), *Classics in Game Theory*.
5. Osborne, M.J. and Rubinstein, A., *A Course in Game Theory*.
6. Owen, G., *Game Theory*.
7. Berger, J.O., *Statistical Decision Theory and Bayesian Analysis*.

Theory of Large Deviations

Prerequisite: Measure Theoretic Probability

- Introduction to large deviations.
- Sanov's theorem and Cramer's theorem for finitely supported random variables.
- General notion of large deviation principle on Polish spaces: Laplace principle, Varadhan's lemma, weak large deviation principle, exponential tightness, goodness of rate function, contraction principle, Bryc's lemma.
- Cramer's theorem for general random variables and vectors.

- Exponential tightness of (a) sample averages of i.i.d. Banach space valued random variables and (b) empirical measures of i.i.d. Polish space valued random variables.

- Cramer's theorem on locally convex separable Hausdorff topological vector spaces. Large deviations of Brownian paths: Schilder's theorem.

- Sanov's theorem on Polish spaces: Donsker-Varadhan variational formula. Gartner and Ellis theorem.

References

1. A. Dembo and O. Zeitouni, *Large Deviations Techniques and Application*.

2. Deuschel and Stroock, *Large Deviations*.

3. Hollander, *Large Deviations*.

4. S.R.S. Varadhan, *Large Deviations and Applications*.

5. P. Dupuis and T. Ellis, *A Weak Convergence Approach to the Theory of Large Deviations*.

Theory of Random Graphs

Prerequisite: Branching Processes

- Review of basic probabilistic tools: First and second moment methods and their variations. The methods of moments. Concentration inequalities for sum of independent Bernoulli variables, binomial and general case. Azuma's inequality. Martingale Convergence Theorem, FKG inequality for finitely many variables. Discrete time branching processes, sub/super/critical phases.

- Classical Random Graphs: Introduction to two basic models of random graphs (Erdős-Rényi random graphs): binomial random graphs and uniform random graphs. Monotonicity property of these graphs. Asymptotic equivalence of the two models.

- Phase transition for the Erdős-Rényi random graphs: the sub-critical, super-critical and critical regimes. Size of the largest sub-critical cluster; size of the largest and second largest super-critical clusters. Size of the largest critical cluster (statement and sketch of the proof only).

- Configuration model: Introduction, erased and repeated models, probability of simplicity. Random regular graphs. Configuration model with i.i.d. degrees.

- Preferential Attachment Models: Definition and examples. Degree of a fixed vertex. Asymptotic of the degree sequence, scale-free networks. Asymptotic of the maximal degree.

- (If time permits) Sub and Super-linear preferential attachment models, asymptotic degree sequence and asymptotic of maximal degree.

References

1. S. Janson, T. Łuczak, A. Riciński: Random Graphs.

2. B. Bollobás: *Random Graphs*.

3. R. van der Hofstad: *Random Graphs and Complex Networks*
   (lecture notes: http://www.win.tue.nl/ rhofstad/NotesRGCN.pdf).